

## 2.3. Chapter 3 Solutions

**Problem 3-1:** 3-1.sen (containing only  $\neg$  and  $\rightarrow$  Between(*c*, *b*, *d*)) and wittgens.wld; Tarski's World Drill  
 The count of negation symbols is odd, so the atomic sentence is negated. Since the atomic sentence is true, this claim is false. As playing of the game proceeds, the number of negation symbols is decremented at each step, and the commitment switches from a true statement to a false statement.

**Problem 3-3:** 3-3.sen (as presented in the text) and wittgens.wld; Tarski's World Drill

Shape of <i>f</i>	Size of <i>f</i>	Number of true sentences in wittgens.wld
Tet	Small	5
Tet	Medium	6
Tet	Large	5
Cube	Small	4
Cube	Medium	4
Cube	Large	4
Dodec	Small	4
Dodec	Medium	4
Dodec	Large	4

At most 6 sentences will be true at once. It would not be possible for all to be simultaneously true, since some pairs, such as 1 & 2 contradict each other. The minimum number of sentences that will be true is 4.

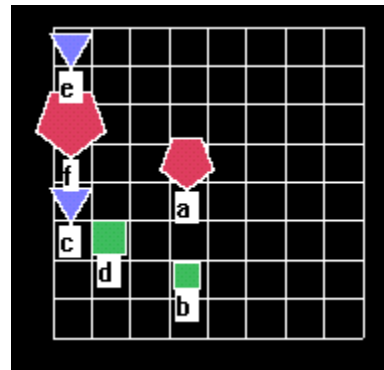
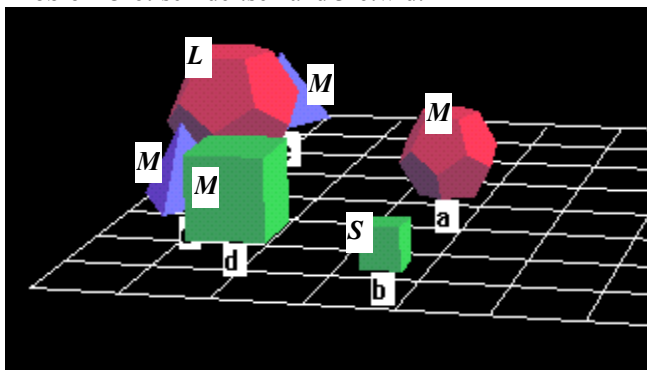
**Problem 3-4:** 3-4.sen (modification of sentences provided in the text) and wittgens.wld; Tarski's World Drill

Shape of <i>f</i>	Size of <i>f</i>	Number of true sentences in wittgens.wld
Tet	Small	6
Tet	Medium	6
Tet	Large	6
Cube	Small	5
Cube	Medium	6
Cube	Large	5
Dodec	Small	5
Dodec	Medium	6
Dodec	Large	5

Again, at most 6 sentences will be simultaneously true. However, the minimum number of sentences that will be true is 5 instead of 4 since only one of two disjuncts must be true but both conjuncts must be true in order to make the corresponding compound sentence true.

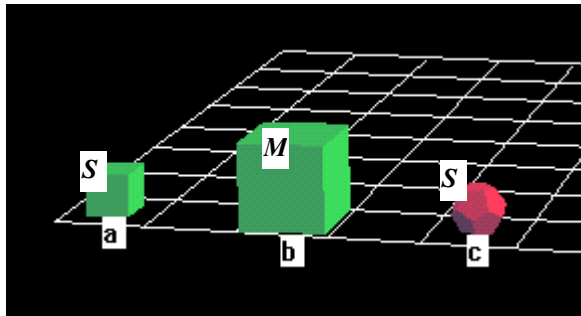
**Problem 3-5:** bool.sen and wittgens.wld; Yes; Tarski's World Drill

**Problem 3-6:** schrder.sen and 3-6.wld:

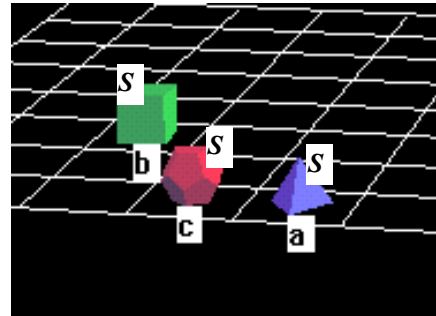


**Problem 3-7:** demorg.sen and 3-7-1.wld and 3-7-2.wld; For each odd numbered sentence  $n$ , sentence  $n+1$  will have the same truth value because each even numbered sentence is logically equivalent to the odd numbered sentence that directly precedes it due to DeMorgan's Laws;

3-7-1.wld:



3-7-2.wld:



**Problem 3-8:** 3-8.sen:

$$1. \neg(\text{Cube}(a) \wedge \text{Larger}(a, b))$$

$$2. \neg\text{Cube}(a) \vee \neg\text{Larger}(a, b)$$

$$3. \neg(\text{Cube}(a) \vee \neg\text{Larger}(b, a))$$

$$4. \neg\text{Cube}(a) \wedge \text{Larger}(b, a)$$

$$5. \neg(\neg\text{Cube}(a) \vee \neg\text{Larger}(a, b) \vee a \neq b)$$

$$6. \text{Cube}(a) \wedge \text{Larger}(a, b) \wedge a = b$$

$$7. \neg(\text{Tet}(b) \vee (\text{Large}(c) \wedge \neg\text{Smaller}(d, e))) \quad ; \quad \neg\text{Tet}(b) \wedge \neg(\text{Large}(c) \wedge \neg\text{Smaller}(d, e))$$

$$8. \neg\text{Tet}(b) \wedge (\neg\text{Large}(c) \vee \text{Smaller}(d, e))$$

$$9. \text{Dodec}(f) \vee \neg(\text{Tet}(b) \vee \neg\text{Tet}(f) \vee \neg\text{Dodec}(f))$$

$$10. \text{Dodec}(f) \vee (\text{Tet}(b) \wedge \text{Tet}(f) \wedge \text{Dodec}(f))$$

**Problem 3-9:**

$$1. (A \wedge B) \wedge A \quad \Leftrightarrow \quad A \wedge B \wedge A$$

$$\Leftrightarrow \quad A \wedge A \wedge B$$

$$\Leftrightarrow \quad A \wedge B$$

$$2. (B \wedge (A \wedge B \wedge C)) \quad \Leftrightarrow \quad (B \wedge A \wedge B \wedge C)$$

$$\Leftrightarrow \quad B \wedge A \wedge B \wedge C$$

$$\Leftrightarrow \quad A \wedge B \wedge B \wedge C$$

$$\Leftrightarrow \quad A \wedge B \wedge C$$

$$3. (A \vee B) \vee (C \wedge D) \vee A \quad \Leftrightarrow \quad (A \vee B) \vee (C \wedge D) \vee A$$

$$\Leftrightarrow \quad A \vee (A \vee B) \vee (C \wedge D)$$

$$\Leftrightarrow \quad A \vee A \vee B \vee (C \wedge D)$$

$$\Leftrightarrow \quad A \vee B \vee (C \wedge D)$$

$$4. (\neg A \vee B) \vee (B \vee C) \quad \Leftrightarrow \quad \neg A \vee B \vee B \vee C$$

$$\Leftrightarrow \quad \neg A \vee B \vee C$$

$$5. (A \wedge B) \vee C \vee (B \wedge A) \vee A \quad \Leftrightarrow \quad (A \wedge B) \vee (B \wedge A) \vee C \vee A$$

$$\Leftrightarrow \quad (A \wedge B) \vee (A \wedge B) \vee C \vee A$$

$$\Leftrightarrow \quad (A \wedge B) \vee C \vee A$$

$$\Leftrightarrow \quad A \vee (A \wedge B) \vee C$$

**Problem 3-10:**

1.  $\neg(\text{Home}(\text{Carl}) \wedge \neg\text{Home}(\text{Claire})) \Leftrightarrow \neg\text{Home}(\text{Carl}) \vee \neg\neg\text{Home}(\text{Claire})$   
 $\Leftrightarrow \neg\text{Home}(\text{Carl}) \vee \text{Home}(\text{Claire})$
2.  $\neg[\text{Happy}(\text{Max}) \wedge (\neg\text{Likes}(\text{Carl}, \text{Claire}) \vee \neg\text{Likes}(\text{Claire}, \text{Carl}))]$   
 $\Leftrightarrow \neg\text{Happy}(\text{Max}) \vee \neg(\neg\text{Likes}(\text{Carl}, \text{Claire}) \vee \neg\text{Likes}(\text{Claire}, \text{Carl}))$   
 $\Leftrightarrow \neg\text{Happy}(\text{Max}) \vee (\neg\neg\text{Likes}(\text{Carl}, \text{Claire}) \wedge \neg\neg\text{Likes}(\text{Claire}, \text{Carl}))$   
 $\Leftrightarrow \neg\text{Happy}(\text{Max}) \vee (\text{Likes}(\text{Carl}, \text{Claire}) \wedge \text{Likes}(\text{Claire}, \text{Carl}))$
3.  $\neg\neg\neg[(\text{Happy}(\text{Max}) \vee \text{Home}(\text{Carl})) \wedge (\text{Happy}(\text{Max}) \vee \text{Happy}(\text{Carl}))]$   
 $\Leftrightarrow \neg[(\text{Happy}(\text{Max}) \vee \text{Home}(\text{Carl})) \wedge (\text{Happy}(\text{Max}) \vee \text{Happy}(\text{Carl}))]$   
 $\Leftrightarrow \neg(\text{Happy}(\text{Max}) \vee \text{Home}(\text{Carl})) \vee \neg(\text{Happy}(\text{Max}) \vee \text{Happy}(\text{Carl}))$   
 $\Leftrightarrow (\neg\text{Happy}(\text{Max}) \wedge \neg\text{Home}(\text{Carl})) \vee (\neg\text{Happy}(\text{Max}) \wedge \neg\text{Happy}(\text{Carl}))$

**Problem 3-11:** 3-11.sen:

- |   |  |
|---|--|
| 1. $\neg\text{FrontOf}(f, a)$                               | 5. $\neg(\text{Larger}(e, c) \vee \text{Larger}(d, c))$  |
| 2. $\text{RightOf}(f, a) \wedge \text{LeftOf}(f, b)$        | 6. $\neg(\text{Larger}(e, d) \vee \text{Smaller}(e, d))$ |
| 3. $\text{BackOf}(f, a) \vee \text{Smaller}(f, a)$          | 7. $\text{Smaller}(c, a) \wedge \text{Larger}(c, e)$     |
| 4. $\text{Between}(e, c, a) \wedge \text{Between}(d, c, a)$ | 8. $\text{FrontOf}(c, f) \wedge \text{Smaller}(c, f)$    |

**Problem 3-12:** 3-12.sen

1.  $\text{Small}(a) \vee (\text{Large}(c) \wedge \text{Large}(d))$   


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2.  $\text{BackOf}(d, b) \wedge \text{BackOf}(e, b)$   


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3.  $(\text{BackOf}(d, b) \wedge \text{BackOf}(e, b)) \wedge (\text{Larger}(d, b) \wedge \text{Larger}(e, b))$   


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4.  $(\text{Cube}(d) \wedge \text{Cube}(c)) \wedge \neg(\text{Small}(d) \vee \text{Small}(c))$   


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5.  $\neg((\text{RightOf}(e, c) \wedge \text{LeftOf}(e, b)) \vee (\text{RightOf}(a, c) \wedge \text{LeftOf}(a, b)))$   


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6.  $\neg\text{Large}(e) \vee \text{BackOf}(e, a)$   


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7.  $\neg\text{Between}(c, a, b) \wedge \neg(\text{FrontOf}(c, a) \vee \text{FrontOf}(c, b))$   


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8.  $(\text{Tet}(a) \wedge \text{Tet}(e)) \vee (\text{Tet}(a) \wedge \text{Tet}(f))$   


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9.  $\neg(\text{FrontOf}(d, c) \vee \text{FrontOf}(d, b)) \wedge \neg(\text{FrontOf}(c, c) \vee \text{FrontOf}(c, b))$   


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10.  $\text{Between}(c, d, f) \vee (\text{Smaller}(c, d) \wedge \text{Smaller}(c, f))$

**Problem 3-13:** Tarski's World Drill**Problem 3-14:** Tarski's World Drill**Problem 3-15:**

1. *Max is a student, not a disk.*  
 $\text{Student}(\text{Max}) \wedge \neg\text{Disk}(\text{Max})$
2. *Claire erased Folly at 2 p.m. and then gave it to Max.*  
 $\text{Erased}(\text{Claire}, \text{Folly}, 2:00) \wedge \text{Gave}(\text{Claire}, \text{Folly}, \text{Max}, 2:00)$
3. *Folly belonged to either Max or Claire at 2:05 p.m.*  
 $\text{Owned}(\text{Max}, \text{Folly}, 2:05) \vee \text{Owned}(\text{Claire}, \text{Folly}, 2:05)$
4. *Neither Max nor Claire erased Folly at 2 p.m. or at 2:05 p.m.*  
 $\neg[\text{Erased}(\text{Max}, \text{Folly}, 2:00) \vee \text{Erased}(\text{Claire}, \text{Folly}, 2:00)] \wedge \neg[\text{Erased}(\text{Max}, \text{Folly}, 2:05) \vee \text{Erased}(\text{Claire}, \text{Folly}, 2:05)]$

**Problem 3-15:** continued

5. *2:00 p.m. is between 1:55 p.m. and 2:05 p.m.*  
 $(1:55 < 2:00) \wedge (2:00 < 2:05)$
6. *When Max gave Folly to Claire at 2 p.m., it wasn't blank, but it was five minutes later.*  
 $\text{Gave}(\text{Max}, \text{Folly}, \text{Claire}, 2:00) \wedge \neg \text{Blank}(\text{Folly}, 2:00) \wedge \text{Blank}(\text{Folly}, 2:05)$

**Problem 3-16:**

- Claire is a student, but Max is not.
- Silly is a disk, and Max did not own it at 2 p.m. OR Silly is a disk that Max did not own at 2 p.m.
- Claire owned Silly or Folly at 2 p.m.
- Max didn't erase both Silly and Folly at 2 p.m.
- When Max gave either Silly or Folly to Claire at 2 p.m., it was blank, but Claire was angry five minutes later.

**Problem 3-17:**

	English	FOL
<b>Names</b>	AIDS, influenza, Abe, Stephen, Sunday, Monday, Dan, George, Al, Bill, Daisy, Dee, Polonius	AIDS, influenza, Abe, Stephen, Sunday, Monday, Dan, George, Al, Bill, Daisy, Dee, Polonius
<b>Predicates</b>	$x$ is less contagious than $y$ $x$ is more deadly than $y$ $x$ is less deadly than $y$ $x$ fooled $y$ on day $d$ $x$ admires $y$ $x$ is a miller $x$ is jolly $x$ lives near $y$ $x$ is a borrower $x$ is a lender $x$ is a river	LessContagious( $x, y$ ) MoreDeadly( $x, y$ ) LessDeadly( $x, y$ ) Fooled( $x, y, d$ ) Admires( $x, y$ ) Miller( $x$ ) Jolly( $x$ ) LivesNear( $x, y$ ) Borrower( $x$ ) Lender( $x$ ) River( $x$ )
<b>Functions</b>	the eldest child of $x$	eldestChild( $x$ )

- $\text{LessContagious}(\text{AIDS}, \text{influenza}) \wedge \text{MoreDeadly}(\text{AIDS}, \text{influenza})$
- $\text{Fooled}(\text{Abe}, \text{Stephen}, \text{Sunday}) \wedge \neg \text{Fooled}(\text{Abe}, \text{Stephen}, \text{Monday})$
- $(\text{Admires}(\text{Dan}, \text{Al}) \wedge \text{Admires}(\text{Dan}, \text{Bill})) \vee (\text{Admires}(\text{George}, \text{Al}) \wedge \text{Admires}(\text{George}, \text{Bill}))$
- $\text{Miller}(\text{Daisy}) \wedge \text{Jolly}(\text{Daisy}) \wedge \text{River}(\text{Dee}) \wedge \text{LivesNear}(\text{Daisy}, \text{Dee})$
- $\neg (\text{Borrower}(\text{eldestChild}(\text{Polonius})) \vee \text{Lender}(\text{eldestChild}(\text{Polonius})))$

**Problem 3-39:**  $\{P \vee (Q \wedge R)\} \vdash (P \vee Q) \wedge (P \wedge R)$

1.	$P \vee (Q \wedge R)$	
2.	$P$	
3.	$P \vee Q$	$\vee$ Intro: 2
4.	$P \vee R$	$\vee$ Intro: 2
5.	$(P \vee Q) \wedge (P \vee R)$	$\wedge$ Intro: 3, 4
6.	$Q \wedge R$	
7.	$Q$	$\wedge$ Elim: 6
8.	$P \vee Q$	$\vee$ Intro: 7
9.	$R$	$\wedge$ Elim: 6
10.	$P \vee R$	$\vee$ Intro: 9
11.	$(P \vee Q) \wedge (P \vee R)$	$\wedge$ Intro: 8, 10
12.	$(P \vee Q) \wedge (P \vee R)$	$\vee$ Elim: 1, 2-5, 6-11

**Problem 3-40:**

1.  $\{P \wedge Q\} \vdash P \vee Q$

1.	$P \wedge Q$	
2.	$P$	$\wedge$ Elim: 1
3.	$P \vee Q$	$\vee$ Intro: 2

2.  $\{(a=b) \wedge (b=c)\} \vdash a=c$

1.	$(a=b) \wedge (b=c)$	
2.	$a=b$	$\wedge$ Elim: 1
3.	$b=c$	$\wedge$ Elim: 1
4.	$a=c$	Ind Id: 2, 3

3.  $\{(A \wedge B) \vee C\} \vdash C \vee B$

1.	$(A \wedge B) \vee C$	
2.	$A \wedge B$	
3.	$B$	$\wedge$ Elim: 2
4.	$C \vee B$	$\vee$ Intro: 3
5.	$C$	
6.	$C \vee B$	$\vee$ Intro: 5
7.	$C \vee B$	$\vee$ Elim: 1, 2-4, 5-6

4.  $\{A, B \vee C\} \vdash (A \wedge B) \vee (A \wedge C)$

1.	$A$	
2.	$B \vee C$	
3.	$B$	
4.	$A \wedge B$	$\wedge$ Intro: 1, 3
5.	$(A \wedge B) \vee (A \wedge C)$	$\vee$ Intro: 4
6.	$C$	
7.	$A \wedge C$	$\wedge$ Intro: 1, 7
8.	$(A \wedge B) \vee (A \wedge C)$	$\vee$ Intro: 7
9.	$(A \wedge B) \vee (A \wedge C)$	$\vee$ Elim: 1, 3-5, 6-8

**Problem 3-41:**  $\{P \vee Q, \neg P\} \vdash Q$

1.	$P \vee Q$	
2.	$\neg P$	
3.	$P$	
4.	$\neg Q$	
5.	$P \wedge \neg P$	$\wedge$ Intro: 3, 2
6.	$\neg\neg Q$	$\neg$ Intro: 4-5
7.	$Q$	$\neg$ Elim: 6
8.	$Q$	
9.	$Q$	Reit: 8
10.	$Q$	$\vee$ Elim: 1, 3-7, 8-9

**Problem 3-42:**  $\emptyset \vdash P \vee \neg P$

1.	$\neg(P \vee \neg P)$	
2.	$P$	
3.	$P \vee \neg P$	$\vee$ Intro: 2
4.	$(P \vee \neg P) \wedge \neg(P \vee \neg P)$	$\wedge$ Intro: 3, 1
5.	$\neg P$	
6.	$\neg P$	
7.	$P \vee \neg P$	$\vee$ Intro: 6
8.	$(P \vee \neg P) \wedge \neg(P \vee \neg P)$	$\wedge$ Intro: 7, 1
9.	$\neg\neg P$	$\neg$ Intro: 6-8
10.	$\neg P \wedge \neg\neg P$	$\wedge$ Intro: 5, 9
11.	$\neg\neg(P \vee \neg P)$	$\neg$ Intro: 1-10
12.	$P \vee \neg P$	$\neg$ Elim: 11

**Problem 3-43:**  $\{P \vee Q, P \vee R\} \vdash P \vee (Q \wedge R)$

1.	$P \vee Q$	
2.	$P \vee R$	
3.	$P$	
4.	$P \vee (Q \wedge R)$	$\vee$ Intro: 3
5.	$Q$	
6.	$P$	
7.	$P \vee (Q \wedge R)$	$\vee$ Intro: 6
8.	$R$	
9.	$Q \wedge R$	$\wedge$ Intro: 5, 8
10.	$P \vee (Q \wedge R)$	$\vee$ Intro: 9
11.	$P \vee (Q \wedge R)$	$\vee$ Elim: 2, 6-7, 8-10
12.	$P \vee (Q \wedge R)$	$\vee$ Elim: 1, 3-4, 5-11

**Problem 3-45:**  $\{Cube(c) \vee Dodec(c), Tet(b)\} \vdash \neg(b=c)$

1.	$Cube(c) \vee Dodec(c)$	
2.	$Tet(b)$	
3.	$b=c$	
4.	$Tet(c)$	Ind Id: 2, 3
5.	$Tet(c) \wedge (Cube(c) \vee Dodec(c))^*$	$\wedge$ Intro: 4, 1
6.	$\neg(b=c)$	$\neg$ Intro: 3-5

\*  $Tet(c) \wedge (Cube(c) \vee Dodec(c))$  is a contradiction since a block cannot be both a tet. and cube (or a dodec.).

**Problem 3-46:**

1.  $\{\neg(P \vee Q)\} \vdash \neg P \wedge \neg Q$

1.	$\neg(P \vee Q)$	
2.	$P$	
3.	$P \vee Q$	$\vee$ Intro: 2
4.	$(P \vee Q) \wedge \neg(P \vee Q)$	$\wedge$ Intro: 3, 1
5.	$\neg P$	$\neg$ Intro: 2-4
6.	$Q$	
7.	$P \vee Q$	$\vee$ Intro: 6
8.	$(P \vee Q) \wedge \neg(P \vee Q)$	$\wedge$ Intro: 3, 1
9.	$\neg Q$	$\neg$ Intro: 6-8
10.	$\neg P \wedge \neg Q$	$\wedge$ Intro: 5, 9