

Problem 5-41:

1. $\{ \exists x \neg P(x) \} \vdash \neg \forall x P(x)$

1. $\exists x \neg P(x)$	
b	
2. $\neg P(b)$	
3. $\forall x P(x)$	\forall Elim: 3
4. $P(b)$	\wedge Intro: 4, 2
5. $P(b) \wedge \neg P(b)$	\neg Intro: 3-5
6. $\neg \forall x P(x)$	\exists Elim: 1, b -6
7. $\neg \forall x P(x)$	

2. $\{ \forall x \neg P(x) \} \vdash \neg \exists x P(x)$

1. $\forall x \neg P(x)$	
2. $\exists x P(x)$	
b	
3. $P(b)$	\forall Elim: 1
4. $\neg P(b)$	\wedge Intro: 3, 4
5. $P(b) \wedge \neg P(b)$	\exists Intro: 6
6. $\exists x (P(x) \wedge \neg P(x))$	\exists Elim: 2, b -6
7. $\exists x (P(x) \wedge \neg P(x))$	
8. $\neg \exists x P(x)$	\neg Intro: 2-7

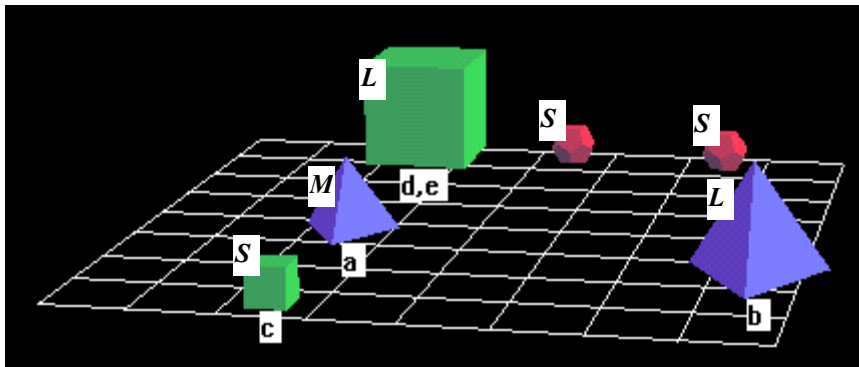
3. $\{ \neg \exists x P(x) \} \vdash \forall x \neg P(x)$

1. $\neg \exists x P(x)$	
2. $\neg \forall x \neg P(x)$	
b	
3. $P(b)$	\exists Intro: 3
4. $\exists x P(x)$	\wedge Intro: 1, 4
5. $\exists x P(x) \wedge \neg \exists x P(x)$	\neg Intro: 3-5
6. $\neg P(b)$	\forall Intro: b -6
7. $\forall x \neg P(x)$	\wedge Intro: 7, 2
8. $\forall x \neg P(x) \wedge \neg \forall x \neg P(x)$	\neg Intro: 2-8
9. $\neg \neg \forall x \neg P(x)$	\neg Elim: 9
10. $\forall x \neg P(x)$	

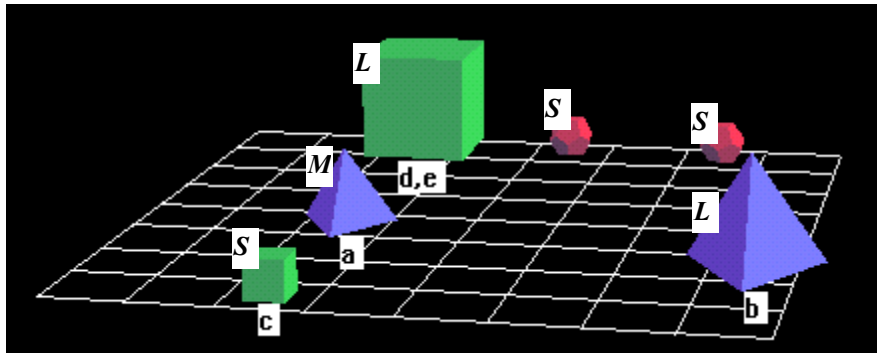
2.6. Chapter 6 Solutions

Problems 6-1: Tarski's World Drill

Problem 6-2: 6-2.wld



Problem 6-3: 6-3.wld:

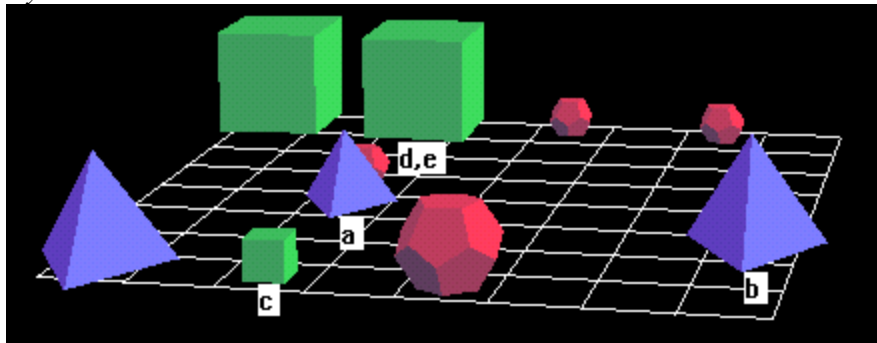


Problem 6-4:

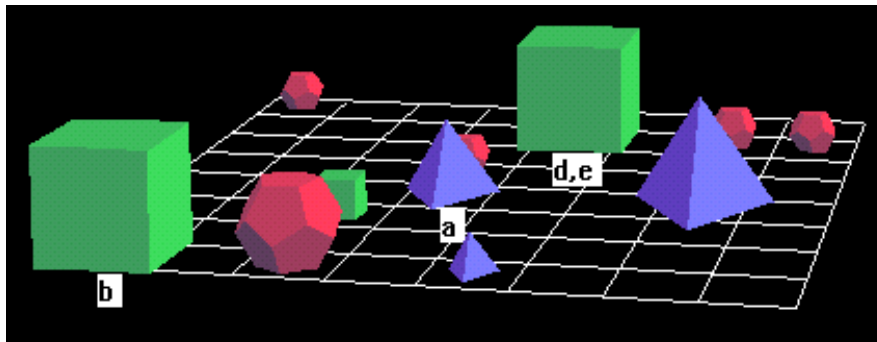
Comments:

- 1-3 cannot be made false by adding objects because they all deal with fixed objects.
- 4-10 cannot be made false by adding objects because they all say that a situation is true in at least one case (only \exists , no \forall). Adding a false case will not change an already existing true case.
- 11 cannot be made false in the existing world since d is in the back row. If it were moved forward a row, an object could be placed behind it to make it false.

My solution:

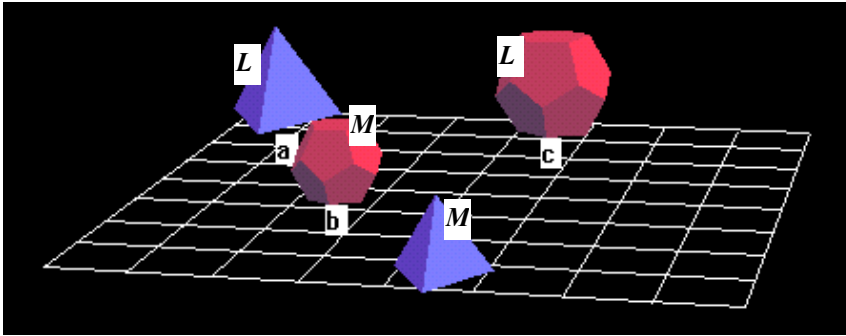


Book's solution:

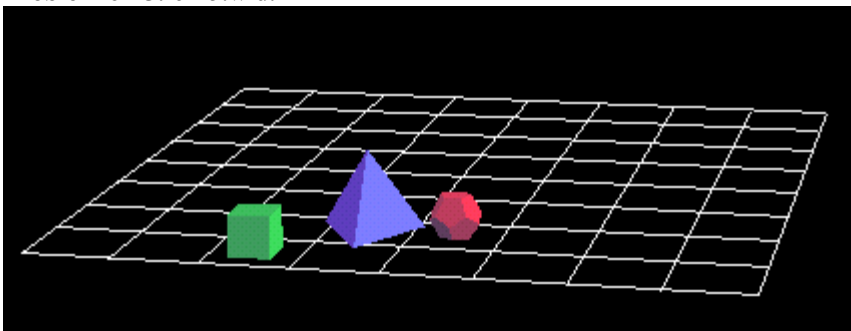


Problem 6-5: Tarski's World Drill

Problem 6-10: Tarski's World Drill

Problem 6-11: 6-11.wld:**Problem 6-13:** Tarski's World Drill

Problem 6-14: Reserved for Logical Project Two (the solution will be provided later)

Problem 6-15: 6-15.wld:**Problem 6-16:** 6-16.sen:

1. $\forall x (Dodec(x) \rightarrow Small(x))$

2. $\exists x (Cube(x) \wedge Medium(x))$

3. $\exists x \exists y (x \neq y \wedge Cube(x) \wedge Cube(y))$

4. $\exists x \exists y \exists z (Tet(x) \wedge Cube(y) \wedge Cube(z) \wedge Between(x, y, z))$

5. $\neg \forall x (Cube(x) \rightarrow \exists y (Dodec(y) \wedge FrontOf(x, y)))$

Problem 6-17: Reserved for Logical Project Two (the solution will be provided later)

Problem 6-18: 6-18.sen:

1. $\forall x (Cube(x) \rightarrow \forall y (Tet(y) \rightarrow LeftOf(x, y)))$
; $\forall x (Cube(x) \rightarrow x\text{-is-to-the-left-of-every-tet})$

2. $\forall x ((Cube(x) \wedge Small(x)) \rightarrow \exists y (Cube(y) \wedge Large(y) \wedge BackOf(x, y)))$
; $\forall x ((Cube(x) \wedge Small(x)) \rightarrow x\text{-is-in-back-of-a-large-cube})$

3. $\exists x (Cube(x) \wedge \forall y (Tet(y) \rightarrow FrontOf(x, y)))$
; $\exists x (Cube(x) \wedge x\text{-is-in-front-of-every-tet})$

4. $\exists x (Cube(x) \wedge Large(x) \wedge \exists y (Cube(y) \wedge Small(y) \wedge FrontOf(x, y)))$
; $\exists x (Cube(x) \wedge Large(x) \wedge x\text{-is-in-front-of-a-small-cube})$

5. $\forall x \neg (\forall y Larger(x, y))$
; $\forall x \neg (x\text{-is-larger-than-everything})$

Problem 6-18: 6-18.sen continued:

6. $\forall x ((\text{Cube}(x) \wedge \forall y(\text{Tet}(y) \rightarrow \text{FrontOf}(x, y))) \rightarrow \text{Large}(x))$
 ; $\forall x ((\text{Cube}(x) \wedge x\text{-is-in-front-of-every-tet}) \rightarrow \text{Large}(x))$
-
7. $\forall x (\exists y(\text{Cube}(y) \wedge \text{Large}(y) \wedge \text{RightOf}(x, y)) \rightarrow \text{Small}(x))$
 ; $\forall x (x\text{-is-to-the-right-of-a-large cube} \rightarrow \text{Small}(x))$
-
8. $\forall x ((\exists y(\text{Cube}(y) \wedge \text{BackOf}(x, y)) \wedge \exists z(\text{Cube}(z) \wedge \text{FrontOf}(x, z))) \rightarrow \neg \text{Large}(x))$
 ; $\forall x ((x\text{-is-in-back-of-a-cube} \wedge x\text{-is-in-front-of-a-cube}) \rightarrow \neg \text{Large}(x))$
-
9. $\forall x (\neg \exists y \text{BackOf}(y, x) \rightarrow \text{Cube}(x))$
 ; $\forall x (\text{there-is-nothing-in-back-of-}x \rightarrow \text{Cube}(x))$
-
10. $\forall x (\text{Dodec}(x) \rightarrow \exists y(\text{Tet}(y) \wedge \text{Smaller}(x, y)))$
 ; $\forall x (\text{Dodec}(x) \rightarrow x\text{-is-smaller-than-some-tet})$

Problem 6-19: Reserved for Logical Project Two (the solution will be provided later)**Problem 6-20:** Reserved for Logical Project Two (the solution will be provided later)**Problem 6-21:** 6-21.sen:

1. $\forall x(\text{Cube}(x) \vee \text{Tet}(x))$
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2. $\forall x(\text{Cube}(x) \rightarrow \forall y(\text{Tet}(y) \rightarrow \text{LeftOf}(x, y)))$; OR $\forall x \forall y ((\text{Cube}(x) \wedge \text{Tet}(y)) \rightarrow \text{LeftOf}(x, y))$
-
3. $\exists x \exists y \exists z (\text{Tet}(x) \wedge \text{Tet}(y) \wedge \text{Tet}(z) \wedge x \neq y \wedge y \neq z \wedge x \neq z)$
-
4. $\forall x ((\text{Cube}(x) \wedge \text{Small}(x)) \rightarrow \exists y(\text{Large}(y) \wedge \text{Cube}(y) \wedge \text{BackOf}(x, y)))$
-
5. $\forall y (\text{Tet}(y) \rightarrow \text{Small}(y))$
-
6. $\forall x (\text{Dodec}(x) \rightarrow \exists y(\text{Tet}(y) \wedge \text{Smaller}(x, y)))$

Problem 6-22: 6-22.sen:

1. $\exists x \exists y (\text{Cube}(x) \wedge \text{Tet}(y) \wedge \neg \text{Smaller}(x, y))$
-
2. $\neg \exists x (\text{Cube}(x) \wedge \exists y \text{RightOf}(x, y))$
-
3. $\exists x \text{Dodec}(x) \vee \exists y \exists z (\text{Large}(y) \wedge \text{Large}(z) \wedge y \neq z)$
-
4. $\forall x ((\text{Cube}(x) \wedge \neg \exists y \text{BackOf}(y, x)) \rightarrow \neg \exists z(\text{Cube}(z) \wedge \text{Larger}(z, x)))$
-
5. $\forall x ((\text{Dodec}(x) \wedge \text{Small}(x)) \rightarrow \exists y \exists z (\text{Cube}(y) \wedge \text{Cube}(z) \wedge \text{Between}(x, y, z)))$
-
6. $\forall x ((\text{Cube}(x) \wedge (\text{Medium}(x) \vee \exists y(\text{Medium}(y) \wedge \text{BackOf}(x, y)))) \rightarrow \forall z (\text{RightOf}(z, x) \rightarrow \text{Tet}(z)))$
-
7. $\forall x \forall y (\text{BackOf}(x, y) \rightarrow \text{Larger}(x, y))$
-
8. $\forall x \exists y (x \neq y \wedge \neg(\text{Larger}(x, y) \vee \text{Smaller}(x, y)))$
-
9. $\forall x (\text{Cube}(x) \rightarrow \exists y(\text{Tet}(y) \wedge \text{RightOf}(y, x) \wedge \neg(\text{Larger}(x, y) \vee \text{Smaller}(x, y))))$
-
10. $\neg \exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \neg(\text{Smaller}(x, y) \vee \text{Larger}(x, y)) \wedge \neg(\text{Smaller}(x, z) \vee \text{Larger}(x, z)))$
-
11. $\forall x \forall y \forall z (\text{Between}(x, y, z) \rightarrow ((\text{Cube}(x) \wedge \text{Cube}(y) \wedge \text{Cube}(z)) \vee (\text{Tet}(x) \wedge \text{Tet}(y) \wedge \text{Tet}(z)) \vee (\text{Dodec}(x) \wedge \text{Dodec}(y) \wedge \text{Dodec}(z))))$

Problem 6-23: 6-23.sen

1. $\neg \exists x (\text{LeftOf}(x, a) \wedge \forall y (\text{LeftOf}(y, b) \rightarrow \text{Larger}(x, y)))$

2. $\forall x \forall y ((\text{LeftOf}(x, a) \wedge \text{LeftOf}(y, b)) \rightarrow \neg \text{Smaller}(x, y))$

3. $\forall x (\text{LeftOf}(x, a) \leftrightarrow \text{LeftOf}(x, b))$

4. $\forall x (\text{LeftOf}(x, a) \rightarrow \exists y ((\forall z ((\text{Cube}(z) \wedge \text{RightOf}(z, b)) \rightarrow \text{BackOf}(y, z))) \wedge \text{Smaller}(x, y)))$

5. $\forall x (\text{Cube}(x) \rightarrow \exists y (\text{Dodec}(y) \wedge \text{Smaller}(x, y))) \wedge \neg \exists x (\text{Cube}(x) \wedge \forall y (\text{Dodec}(y) \rightarrow \text{Smaller}(x, y)))$

6. $\forall x ((\text{Cube}(x) \wedge \text{Larger}(a, x)) \rightarrow \forall y (\text{Tet}(y) \rightarrow \text{Smaller}(a, y)))$

7. $\forall x (\forall y (x \neq y \rightarrow \text{Larger}(x, y)) \rightarrow \text{Dodec}(x))$

8. $\forall x (\neg \exists y \text{FrontOf}(y, x) \rightarrow \text{Tet}(x))$

9. $\neg \exists x \exists y \exists z (\text{Between}(x, y, z) \wedge ((\text{Tet}(y) \wedge \text{Tet}(z)) \vee (\text{Cube}(y) \wedge \text{Cube}(z)) \vee (\text{Dodec}(y) \wedge \text{Dodec}(z))))$

10. $\forall x \forall y \forall z (\text{Between}(x, y, z) \rightarrow \text{Cube}(x))$

11. $\exists x (\text{BackOf}(x, b) \wedge \exists y \exists z (\text{BackOf}(y, x) \wedge \text{BackOf}(z, x) \wedge y \neq z))$

12. $\exists x (\text{Larger}(x, b) \wedge \exists y \exists z (\text{Smaller}(y, x) \wedge \text{Smaller}(z, x) \wedge y \neq z))$

Problem 6-24:

1. $\forall x (\text{Student}(x) \rightarrow \exists y \exists z (\text{Student}(z) \wedge \text{Disk}(y) \wedge \exists t \text{Gave}(x, y, z, t)))$
2. $\text{Student}(\text{Claire}) \vee \exists x \exists t (\text{Disk}(x) \wedge \text{Owned}(\text{Claire}, x, t))$
It is assumed that this sentence was asserted at 2 p.m. on January 2, 1996.
3. $\neg \exists x \exists t (\text{Person}(x) \wedge \text{Owned}(x, \text{Folly}, t) \wedge \text{Owned}(x, \text{Silly}, t))$
4. $\neg \exists x (\text{Student}(x) \wedge \forall y (\text{Disk}(y) \rightarrow \exists t \text{Erased}(x, y, t)))$
5. $\neg \exists x \exists y (\text{Person}(x) \wedge \text{Disk}(y) \wedge \text{Owned}(x, y, 2:00) \wedge \text{Angry}(x, 2:00))$
6. $\neg \exists x \exists y \exists t (\text{Person}(x) \wedge \text{Disk}(y) \wedge (t < 12:00) \wedge \text{Gave}(x, y, \text{Claire}, t))$
7. $\forall x \forall t ((\text{Disk}(x) \wedge \text{Gave}(\text{Max}, x, \text{Claire}, t)) \rightarrow (\text{Owned}(\text{Claire}, x, t) \wedge \neg \text{Owned}(\text{Max}, x, t)))$
8. $\neg \exists x \exists y \exists t (\text{Person}(x) \wedge \neg \text{Owned}(x, y, t) \wedge \exists z (\text{Person}(z) \wedge \text{Give}(x, y, z, t)))$
9. $\forall x \forall y \forall t \forall z ((\text{Disk}(x) \wedge \text{Owned}(\text{Max}, x, t) \wedge \text{Erased}(\text{Max}, x, t)) \rightarrow (\text{Disk}(y) \wedge \text{Owned}(\text{Claire}, y, z) \wedge \text{Erased}(\text{Claire}, y, z) \wedge t < z))$
10. $\exists x \exists t (\text{Disk}(x) \wedge (2:00 < t) \wedge (t < 3:00) \wedge \text{Gave}(\text{Max}, x, \text{Claire}) \wedge \text{Blank}(x, t))$
11. $\neg \exists x \exists y \exists z \exists t (\text{Student}(x) \wedge \text{Disk}(y) \wedge \text{Disk}(z) \wedge y \neq z \wedge \text{Owned}(x, y, t) \wedge \text{Owned}(x, z, t))$
12. $\forall x \exists t \exists t' ((\text{Student}(x) \wedge \exists y \exists z \exists t (\text{Student}(x) \wedge \text{Disk}(y) \wedge \text{Disk}(z) \wedge y \neq z \wedge \text{Owned}(x, y, t') \wedge \text{Owned}(x, z, t') \wedge t \leq t')) \rightarrow \exists y \exists z (\text{Disk}(y) \wedge \text{Disk}(z) \wedge y \neq z \wedge \text{Owned}(\text{Claire}, y, t) \wedge \text{Owned}(\text{Claire}, z, t)))$
13. $\forall x \forall y ((\text{Person}(x) \wedge \text{Disk}(y) \wedge \forall t \text{Owned}(x, y, t)) \rightarrow \exists z \text{Erased}(x, y, z))$
14. $\forall x \forall y \forall t ((\text{Person}(x) \wedge \text{Disk}(y) \wedge \text{Owned}(x, y, t)) \rightarrow \exists z (\text{Owned}(x, y, z) \wedge \text{Erased}(x, y, z)))$
15. $\forall x \forall t [\exists y (\text{Person}(y) \wedge \text{Erased}(y, x, t)) \rightarrow (\text{Disk}(x) \wedge \neg \text{Blank}(x, t))]$

Problem 6-25:

1. Every student owns a disk.
2. Some student owns every disk.
3. Claire gave someone everything that Max ever gave to her.
4. Claire owns something that Max gave her earlier.
5. Max gave Claire something in the last five minutes.
6. Max gave something to everyone in the last five minutes.
7. Some student gave everyone something in the last five minutes.

Problem 6-26:

	English	FOL
Names	you I Dee morn night	You I Dee Morn Night
Predicates	<i>x</i> is a sucker <i>x</i> was born at <i>t</i> <i>x</i> is a place <i>x</i> goes to <i>y</i> at <i>t</i> <i>x</i> is a soothsayer <i>x</i> is a truthsayer <i>x</i> makes a better living than <i>y</i> <i>x</i> gave <i>y</i> to <i>z</i> <i>x</i> is required of <i>z</i> <i>x</i> does right at <i>t</i> <i>x</i> is a person <i>x</i> gratifies <i>y</i> <i>x</i> astonishes <i>y</i> <i>x</i> is a miller <i>x</i> is jolly <i>x</i> is a river <i>x</i> lived near <i>y</i> at <i>t</i> <i>x</i> is less than or equal to <i>y</i> <i>x</i> worked at <i>t</i> <i>x</i> sang at <i>t</i> <i>x</i> is a lark <i>x</i> is more blithe than <i>y</i> <i>x</i> is company at <i>t</i> <i>x</i> should expect <i>y</i> at <i>t</i> <i>x</i> blushes <i>x</i> needs to blush <i>x</i> is an animal <i>x</i> is man <i>x</i> can fool <i>y</i> at <i>t</i> <i>x</i> loves <i>y</i>	Sucker(<i>x</i>) Born(<i>x</i> , <i>t</i>) Place(<i>x</i>) Go(<i>x</i> , <i>y</i> , <i>t</i>) Soothsayer(<i>x</i>) Truthsayer(<i>x</i>) MakeBetterLiving(<i>x</i> , <i>y</i>) Gave(<i>x</i> , <i>y</i> , <i>z</i>) Required(<i>x</i> , <i>y</i>) DoRight(<i>x</i> , <i>t</i>) Person(<i>x</i>) Gratify(<i>x</i> , <i>y</i>) Astonish(<i>x</i> , <i>y</i>) Miller(<i>x</i>) Jolly(<i>x</i>) River(<i>x</i>) LivedNear(<i>x</i> , <i>y</i> , <i>t</i>) $x \leq y$ Worked(<i>x</i> , <i>t</i>) Sang(<i>x</i> , <i>t</i>) Lark(<i>x</i>) MoreBlithe(<i>x</i> , <i>y</i>) Company(<i>x</i> , <i>t</i>) ShouldExpect(<i>x</i> , <i>y</i> , <i>t</i>) Blushes(<i>x</i>) NeedsToBlush(<i>x</i>) Animal(<i>x</i>) Man(<i>x</i>) CanFool(<i>x</i> , <i>y</i> , <i>t</i>) Love(<i>x</i> , <i>y</i>)
Functions		

- $\forall t \exists x (Sucker(x) \wedge Born(x, t))$
- $\forall x \forall t ((Place(x) \wedge Go(You, x, t)) \rightarrow Go(I, x, t))$
- $\forall x \forall y ((Soothsayer(x) \wedge Truthsayer(y)) \rightarrow MakeBetterLiving(x, y))$
- $\forall x (\neg \exists y \exists z Gave(z, y, x) \rightarrow \neg \exists z Required(z, x))$
- $\forall t DoRight(You, t) \rightarrow \{\exists x (Person(x) \wedge Gratify(You, x)) \wedge \forall x [(Person(x) \wedge \neg Gratify(You, x)) \rightarrow Astonish(You, x)]\}$
- $\exists t \exists x \{Miller(x) \wedge Jolly(x) \wedge River(Dee) \wedge LivedNear(x, Dee, t) \wedge \forall z [((Morn \leq z) \wedge (z \leq Night)) \rightarrow (Worked(x, z) \wedge Sang(x, z))] \wedge \forall y [Lark(y) \rightarrow \neg MoreBlithe(y, x)]\}$
- $\forall t \forall x (Company(x, t) \rightarrow ShouldExpect(You, x, t))$
- $\forall x (((Blushes(x) \vee NeedsToBlush(x)) \wedge Animal(x)) \rightarrow Man(x))$
- $\forall x (Person(x) \rightarrow [\forall y (Person(y) \rightarrow \exists t CanFool(x, y, t)) \wedge \exists z (Person(z) \wedge \forall u CanFool(x, z, u)) \wedge \neg \forall y \forall t (Person(y) \rightarrow CanFool(x, y, t))])$
- $\forall x [(Person(x) \wedge \exists y (Person(y) \wedge Love(x, y))) \rightarrow \forall z (Person(z) \rightarrow Love(z, x))]$

Problem 6-32:

	English	FOL
Names	Claire Max Melanie Mary	Claire Max Melanie Mary
Predicates	x is the mother of y x is older than y	MotherOf(x, y) Older(x, y)
Functions	x 's mother	mother(x)

- Older(mother(Claire), mother(Max))
 $\exists x \exists y [\text{MotherOf}(x, \text{Claire}) \wedge \text{MotherOf}(y, \text{Max}) \wedge \forall z (\text{MotherOf}(z, \text{Claire}) \rightarrow x = z) \wedge \forall w (\text{MotherOf}(w, \text{Max}) \rightarrow y = w) \wedge x \neq y \wedge \text{Older}(x, y)]$
- $\forall x \text{Older}(\text{mother}(\text{mother}(x)), \text{Melanie})$
 $\forall x \exists y \exists z [\text{MotherOf}(y, x) \wedge \text{MotherOf}(z, y) \wedge x \neq y \wedge y \neq z \wedge x \neq z \wedge \forall w [\text{MotherOf}(w, x) \rightarrow y = w] \wedge \forall w [\text{MotherOf}(w, y) \rightarrow z = w] \wedge \text{Older}(z, \text{Melanie})]$
- $\exists x \text{Older}(\text{Mary}, \text{mother}(\text{mother}(x)))$
 $\exists x \exists y \exists z [\text{MotherOf}(x, y) \wedge \text{MotherOf}(z, y) \wedge x \neq y \wedge x \neq z \wedge y \neq z \wedge \forall w (\text{MotherOf}(w, x) \rightarrow y = w) \wedge \forall w (\text{MotherOf}(w, y) \rightarrow z = w) \wedge \text{Older}(\text{Mary}, z)]$

Problem 6-33:

- $[\text{height}(\text{father}(\text{Max})) > \text{height}(\text{Max})] \wedge \neg[\text{height}(\text{father}(\text{Max})) > \text{height}(\text{father}(\text{Claire}))]$
- $\exists x [\text{height}(x) > \text{height}(\text{father}(\text{Claire}))]$
- $\forall x \exists y [\text{height}(x) > \text{height}(y)]$
- $\neg \exists x (\text{height}(x) > \text{height}(x))$
- $\forall x [(\text{height}(x) > \text{height}(\text{Claire})) \rightarrow (\text{height}(x) > \text{height}(\text{Max}))]$
- $\forall x \{(\text{height}(\text{Claire}) > \text{height}(x)) \rightarrow \exists y[(\text{height}(y) > \text{height}(x)) \wedge (\text{height}(\text{father}(\text{Max})) > \text{height}(y))]\}$

2.7. Chapter 7 Solutions

Problem 7-1: Tarski's World Drill**Problem 7-2:**

- There is exactly one Tove.
- There is at most one Tove.
- There is exactly one Tove.
- There is at most one Tove.
- Objects are identical iff they are Toves.

Problem 7-3: Reserved for Logical Project Three (the solution will be provided later)

Problem 7-4: Reserved for Logical Project Three (the solution will be provided later)

Problem 7-5: Reserved for Logical Project Three (the solution will be provided later)