Problem 3. Translate the following English sentences into the formal language of *the Tarski's World* (50 points).

(1) Either *a* is smaller than *b* or both *a* and *b* are larger than *c*.

(2) *a* and *b* are both in front of *c*; moreover, both are smaller than it.

(3) *c* is neither between *a* and *b*, nor in front of either of them.

(4) Neither *d* nor *c* is in front of either *b* or *e*.

(5) **Only one** block, either *a* or *b*, is between *c* and *d*.

(6) **Extra credit**: Although a small cube *a* is in front of a large dodecahedron *b*, the former is to the left of the latter unless *a* is between two tetrahedrons *c* and *d*.

3.1.3. Quiz Three

Problem 1: Using the names and predicates presented in Table 1 on page 23, translate the following into FOL (50 points = 10 points × 5).

1. If Folly belonged to Max at 2 p.m., then it was not blank 5 minute later.
2. Claire erased Folly at 2 p.m. if and only if Max did not erase Folly at 2 p.m..
3. Folly was blank at 2 p.m. if Claire or Max erased it at 1:55 p.m..
4. Folly was blank at 2 p.m. unless Claire did not erase it at 1:55 p.m..
5. Folly was not blank at 2 p.m. only if neither Claire nor Max erased it 10 minutes before.

Problem 2: Give formal proofs of the following (50 points = 25 points × 2).

1. $\{ A \rightarrow B,\ A \rightarrow (B \rightarrow C),\ B \rightarrow (C \rightarrow D) \} \models A \rightarrow D$
2. $\{ (F \wedge G) \leftrightarrow H,\ F \rightarrow G \} \models F \leftrightarrow H$

3.2. Solutions to Quizzes

3.2.1. Quiz One Solutions

Problem 1

1. Gave (Claire, Silly, Max, 3:05)
2. Angry (Max, 2:10)
3. Owen (Max, Folly, 3:00)
4. 3:00 < 3:01
5. Erased (Max, Folly, 2:00)

Problem 2
A. Translation manual

	English	FOL
Names	Claire, Max, John, Nancy Symbolic Logic, Class 2000 2, 6, 11	Claire, Max, John, Nancy SymLogic, Class2000 2, 6, 11
Functions	the best student of x the girl friend of x the best friend of x the youngest son of x and y the oldest daughter of x and y the sum of x and y the next number of x	bestS (x) girlF (x) bestF(x) youngestS(x, y) oldestD (x, y) sum (x, y) nextN(x)
Predicates	x is a good student in y x is y x is taller than y x prefers y to z x is less than y	GStudent(x, y) x = y Taller (x, y) Prefer (x, y, z) Less (x, y)

B. Translations
1. GStudent (Claire, SymLogic)
2. Claire = bestS (Class2000)
3. Taller (Claire, girlF (bestF (Max)))
4. Prefer (Max, Claire, girlF(John))
5. Taller (oldestD(John, Nancy), youngestS(John, Nancy))
6. Less (sum (2, 6), nextN(11))

3.2.2. Quiz Two Solutions

Problem 1.
Proof:

1.	Larger (b, a)	
2.	c = b	
3.	a = d	
4.	Larger (b, d)	Ind. Id: 1, 3
5.	c = c	Refl =
6.	b = c	Ind. Id: 5, 2
7.	Larger (c, d)	Ind. Id: 4, 6

Problem 2.
(1) $\neg [(\neg A \wedge \neg \neg B) \vee \neg (A \vee C)] \Leftarrow ? \Rightarrow (C \vee A) \wedge (\neg B \vee A)$
Proof:
$\neg [(\neg A \wedge \neg \neg B) \text{ v } \neg (A \vee C)]$
$\Leftrightarrow \quad \neg(\neg A \wedge \neg\neg B) \wedge \neg\neg (A \vee C)$
$\Leftrightarrow \quad (\neg\neg A \vee \neg\neg\neg B) \wedge (A \vee C)$
$\Leftrightarrow \quad (A \vee \neg B) \wedge (A \vee C)$
$\Leftrightarrow \quad (C \vee A) \wedge (\neg B \vee A)$, as desired.

(2) ¬ [(¬ A v B) v ¬ (A ∧ ¬ (A v B))] ⇐?⇒ A ∧ ¬ (A v B)

Proof:

¬ [(¬ A v B) v ¬ (A ∧ ¬ (A v B))]
⇔ ¬(¬ A v B) ∧ ¬¬(A ∧ ¬ (A v B))
⇔ (¬¬A ∧¬B) ∧ (A ∧ ¬ (A v B))
⇔ (A ∧¬B) ∧ A ∧ (¬ A ∧ ¬ B)
⇔ ~~A ∧¬B ∧ A ∧ ¬ A ∧¬ B~~
⇔ A ∧ ¬A ∧ ¬B
⇔ A ∧ ¬ (A ∨ B) , as desired.

Problem 3. Translate the following English sentences into the formal language of *the Tarski's World* (50 points).

(1) Smaller(a, b) ∨ (Larger(a, c) ∧ Larger (b, c))

(2) (FrontOf(a, c) ∧ FrontOf(b, c)) ∧ (Smaller(a, c) ∧ Smaller(b, c))

(3) ¬Between (c, a, b) ∧ ¬ (FrontOf(c, a) ∨ FrontOf(c, b))

(4) ¬(FrontOf(d, b) ∨ FrontOf(d, e)) ∧¬(FrontOf(c, b) ∨ FrontOf(c, e))

(5) (Between (a, c, d) ∨ Between (b, c, d)) ∧ ¬ (Between (a, c, d) ∧ Between (b, c, d))

(6) [(Small(a) ∧ Cube(a) ∧ Large(b) ∧ Dodec(b) ∧ FrontOf(a, b)] ∧ [LeftOf(a, b) ∨ (Tet(c) ∧ Tet(d) ∧ Between(a , c, d))]

3.2.3. Quiz Three Solutions

Problem 1

1. *If Folly belonged to Max at 2 p.m., then it was not blank 5 minute later.*
   Owned (Max, Folly, 2:00) → ¬ Blank (Folly, 2:05)
2. *Claire erased Folly at 2 p.m. if and only if Max did not erase Folly at 2 p.m..*
   Erased (Claire, Folly, 2:00) ↔ ¬ Erased (Max, Folly, 2:00)
3. *Folly was blank at 2 p.m. if Claire or Max erased it at 1:55 p.m.*
   (Erased (Claire, Folly, 1:55) v Erased (Max, Folly, 1:55)) → Blank (Folly, 2:00)
4. *Folly was blank at 2 p.m. unless Claire did not erase it at 1:55 p.m..*
   Blank (Folly, 2:00) v ¬ Erased (Cliare, Folly, 1:55)

   Or Erased (Cliare, Folly, 1:55) → Blank (Folly, 2:00)
5. *Folly was not blank at 2 p.m. only if neither Claire nor Max erased it 10 minutes before.*
   ¬ Blank (Folly, 2:00) → ¬ (Erased (Claire, Folly, 1:50) v Erased (Max, Folly, 1:50))

Problem 2

1. { A → B, A → (B → C), B → (C → D) } |= A → D

1.	A → B	
2.	A → (B → C)	
3.	B → (C → D)	
	4. A	
	5. B	→Elim: 4, 1
	6. B → C	→Elim: 4, 2
	7. C → D	→Elim: 5, 3
	8. C	→Elim: 5, 6
	9. D	→Elim: 8, 7
10.	A → D	→Intro: 4-9

2. { (F ∧ G) ↔ H, F → G } |= F ↔ H

1.	(F ∧ G) ↔ H	
2.	F → G	
3.	F	
4.	G	→Elim: 3, 2
5.	F ∧ G	∧Intro: 3, 4
6.	H	↔Elim: 5, 1
7.	H	
8.	F ∧ G	↔Elim: 7, 1
9.	F	∧Elim: 8
10.	H ↔ G	↔Intro: 3-6, 7-9.

3.3. Tests (Solutions follow in 3.4)

3.3.1. Test One

Problem 1: Use Double Negation rule, DeMorgen rules and any other rules to prove that the following pair of sentences are logically equivalent (20 points).

¬ (A v ¬ (B ∧ C)) ∧ ¬ (¬ B v (A v B)) ⇐?⇒ (C ∧ B) ∧ ¬ (B v A)

Problem 2: By creating your own translation manual, translate the following English sentences into FOL (40 points).

(1) Max can marry either Nancy's oldest daughter or her youngest daughter (in a monogamy society).

(2) Jenny is Nancy's youngest daughter and Claire is her oldest daughter.

(3) Neither Claire nor Jenny is in love with Max.

(4) Jenny will not marry Max unless he is intelligent and in love with her.

(5) Max is not both intelligent and in love with Jenny.

Problem 3: Give formal proofs of the followings (40 points).

(1) { (A ∧ C) v (D ∧ B) } |= C v B (about 8 steps)

(2) { (A ∧ C) v (D ∧ C), B } |= C ∧ B (about 10 steps)

(3) { A v ¬ B, ¬ A } |= ¬ B (about 9 steps)

(4) { ¬ (P v Q) } |= ¬ P ∧ ¬Q (about 10 steps)

(5) **Bonus** (up to 5 points)
{(Small (a) ∧ Smaller (a, b)) v (Large (b) ∧ Smaller (a, b)), c = b} |= Smaller (a, c) ∧ c = b (about 11 steps)