2. { (F ∧ G) ↔ H, F → G } |= F ↔ H

1.	(F ∧ G) ↔ H	
2.	F → G	
3.	F	
4.	G	→Elim: 3, 2
5.	F ∧ G	∧Intro: 3, 4
6.	H	↔Elim: 5, 1
7.	H	
8.	F ∧ G	↔Elim: 7, 1
9.	F	∧Elim: 8
10.	H ↔ G	↔Intro: 3-6, 7-9.

3.3. Tests (Solutions follow in 3.4)

3.3.1. Test One

Problem 1: Use Double Negation rule, DeMorgen rules and any other rules to prove that the following pair of sentences are logically equivalent (20 points).

¬ (A v ¬ (B ∧ C)) ∧ ¬ (¬ B v (A v B)) ⇐?⇒ (C ∧ B) ∧ ¬ (B v A)

Problem 2: By creating your own translation manual, translate the following English sentences into FOL (40 points).

(1) Max can marry either Nancy's oldest daughter or her youngest daughter (in a monogamy society).

(2) Jenny is Nancy's youngest daughter and Claire is her oldest daughter.

(3) Neither Claire nor Jenny is in love with Max.

(4) Jenny will not marry Max unless he is intelligent and in love with her.

(5) Max is not both intelligent and in love with Jenny.

Problem 3: Give formal proofs of the followings (40 points).

(1) { (A ∧ C) v (D ∧ B) } |= C v B (about 8 steps)

(2) { (A ∧ C) v (D ∧ C), B } |= C ∧ B (about 10 steps)

(3) { A v ¬ B, ¬ A } |= ¬ B (about 9 steps)

(4) { ¬ (P v Q) } |= ¬ P ∧ ¬Q (about 10 steps)

(5) **Bonus** (up to 5 points)
{(Small (a) ∧ Smaller (a, b)) v (Large (b) ∧ Smaller (a, b)), c = b} |= Smaller (a, c) ∧ c = b (about 11 steps)

3.3.2. Test Two

Problem 1. Translate the following English sentences into FOL (using the language of Tarski's World; Domain: all the blocks in a Tarski's world). (40 points)

1. If *b* is neither to the right nor left of *d*, then at least one of them is a cube.
2. *b* and *c* are the same size if and only if *b* is a tetrahedron and *c* is a dodecahedron.
3. *a* and *c* are both cubes only if exactly one of them is small.
4. Any cube in front of *a* is larger than *a*.
5. Cube *b* is in front of some small dodecahedron.
6. *a* is not in back of every medium tetrahedron.
7. *b* is not between *a* and any cubes.
8. Only cubes are smaller than *b*.
9. *b* is larger than nothing but cubes.
10. If some cube is in front of *a*, then it is in back of *b*.

Problem 2. Prove the following pair of sentences are logically equivalent (14 points).

$\neg [\exists y\ (Tet(y) \wedge Large\ (y)) \wedge \forall y\ (Tet(y) \rightarrow \neg\ Large(y))\ ]$
$\Leftarrow ? \Rightarrow \forall x\ (Tet(x) \rightarrow \neg\ Large(x))\ \ v\ \exists y\ (Tet(y) \wedge Large(y))$

Problem 3. For the following sentences in FOL, translate them into colloquial English sentences (16 points)

(1) $\forall x\ [\ (Small(x) \wedge Cube(x)) \rightarrow BackOf(x, a)\ ]$

(2) $\exists x\ [\ (Dodec(x) \wedge Large(x)) \wedge \exists y\ (Cube(y) \wedge RightOf(x, y))\ ]$

(3) $\neg\ \forall x\ (\ Cube(x) \rightarrow LeftOf(x, a)\ )$

(4) $\neg\ \exists x\ [\ Dodec(x) \wedge Large(x) \wedge BackOf(x, a)\ ]$

Problem 4. Give formal proofs of the followings (30 points)

(1) $\{\ \forall x\ (Cube(x) \rightarrow Small(x)),\quad \forall x\ (Small(x) \rightarrow BackOf(x, b))\ \} \models \forall x\ (Cube(x) \rightarrow BackOf(x, b))$

(2) $\{\ \forall x\ (Small(x) \rightarrow Cube(x)),\quad \exists x \neg Cube(x)\ \} \models \exists x \neg Small(x)$

3.3.3. Test Three

Problem 1: Translate the following English sentences into *the language of Tarski's World*.
DD: all blocks in a Tarski's world. (60 points = 15 × 4)

(1) Some block is to the right of every other block.

(2) Every block is to the right of some other block.

(3) If some block is a cube, then it is to the left of some dodecahedron.

(4) *Only* small tetrahedrons are in front of all cubes.

(5) Anything with *nothing* in front of it is small.

(6) Not all cubes are in front of some small tetrahedron.

(7) Some tetrahedron is as large as some cube.

(8) Some cube is the largest block.

(9) All cubes but *a* are in front of some dodecahedron.

(10) Some block but cubes is in back of all tetrahedrons.

(11) There are at least two cubes in front of *a*.

(12) There is at most one small cube.

(13) There is exactly one cube in front of *b*.

(14) There is exactly one cube and it is in front of *b*.

(15) The cube is in front of a dodecahedron and in back of a tetrahedron.

Problem 2: For the following sentences in FOL, (a) translate them into colloquial English sentences, and (b) assess their truth values on the basis of the world attached (40 points = 8 × 5).

_____ (1) $\exists x \forall y\ [\ Large(x) \wedge (Small(y) \rightarrow BackOf(x, y)\ )\ ]$

_____ (2) $\forall y \exists x\ [Small(y) \rightarrow (Large(x) \wedge FrontOf(y, x))\ ]$

_____ (3) $\neg\exists x\ \{Cube\ (x) \wedge \forall y\ [(Tet(y) \wedge Large(y)) \rightarrow LeftOf\ (x, y)]\ \}$

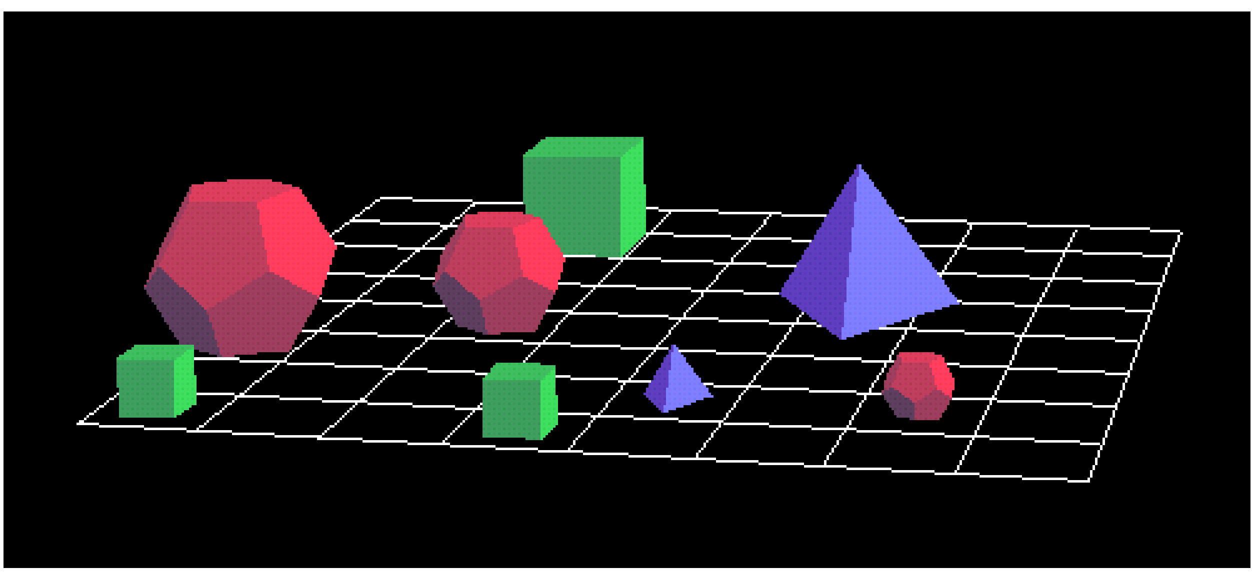
_____ (4) $\neg\forall x\ [\ Cube(x) \rightarrow \exists y\ (Dodec\ (y) \wedge FrontOf\ (x, y))]$

_____ (5) $\forall x \forall y\ [(Tet(x) \wedge Tet(y) \wedge Large(x) \wedge Large(y)) \rightarrow y = x]$

_____ (6) $\exists x \exists y\ [Tet(x) \wedge Tet(y) \wedge x \neq y \wedge \forall z\ (Tet(z) \rightarrow (z = x \vee z = y))]$

_____ (7) $\exists x\ [Tet(x) \wedge \forall y\ (Tet(y) \rightarrow y = x) \wedge \exists z\ (Large(z) \wedge Dodec(z) \wedge BackOf(x, z))]$

_____ (8) $\exists x\ \{Tet(x) \wedge \exists y\ (Large(y) \wedge Dodec(y) \wedge BackOf(x, y)) \wedge$
$\forall z\ [(Tet(z) \wedge \exists y\ (Large(y) \wedge Dodec(y) \wedge BackOf(z, y))) \rightarrow z = x]\}$



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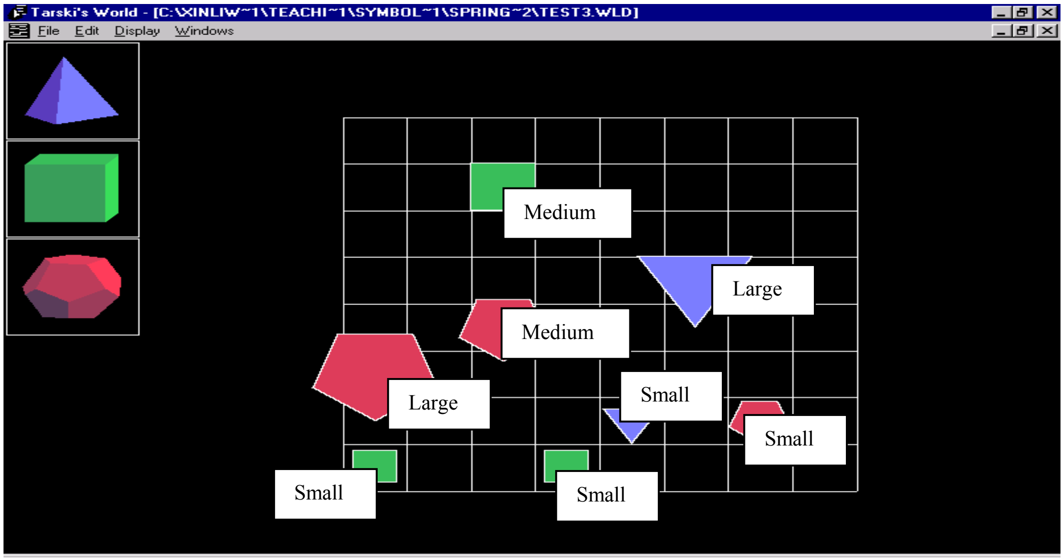

3.4. Solutions to Tests

3.4.1. Test One Solutions

Problem 1.

$$\neg (A \vee \neg (B \wedge C)) \wedge \neg (\neg B \vee (A \vee B))$$
$$\Leftrightarrow \neg A \wedge \neg\neg (B \wedge C) \wedge \neg\neg B \wedge \neg (A \vee B)$$
$$\Leftrightarrow \neg A \wedge B \wedge C \wedge B \wedge \neg A \wedge \neg B$$
$$\Leftrightarrow \neg A \wedge B \wedge C \wedge \neg B$$
$$\Leftrightarrow (C \wedge B) \wedge (\neg B \wedge \neg A)$$
$$\Leftrightarrow (C \wedge B) \wedge \neg (B \vee A)$$

Problem 2.
A translation manual

	English	FOL
Names	Max, Claire, Jenny, Nancy	same
Predicates	x marry y x is y x is in love with y x is intelligent.	Marry (x, y) x = y InLove (x, y) Intelligent (x)
Functions	the oldest daughter of x the youngest daughter of x	o-daughter (x) y-daughter (x)

Translations

(1) [Marry (Max, o-daughter(Nancy)) ∨ Marry (Max, y-daughter (Nancy))] ∧
¬ [Marry (Max, o-daughter(Nancy)) ∧ Marry (Max, y-daughter (Nancy))] exclusive sense of OR!

(2) Jenny = y-daughter (Nancy) ∧ Claire = y-daughter (Nancy)

(3) ¬ InLove(Claire, amx) ∧ ¬ InLOve(Jenny, Max) complete denial!