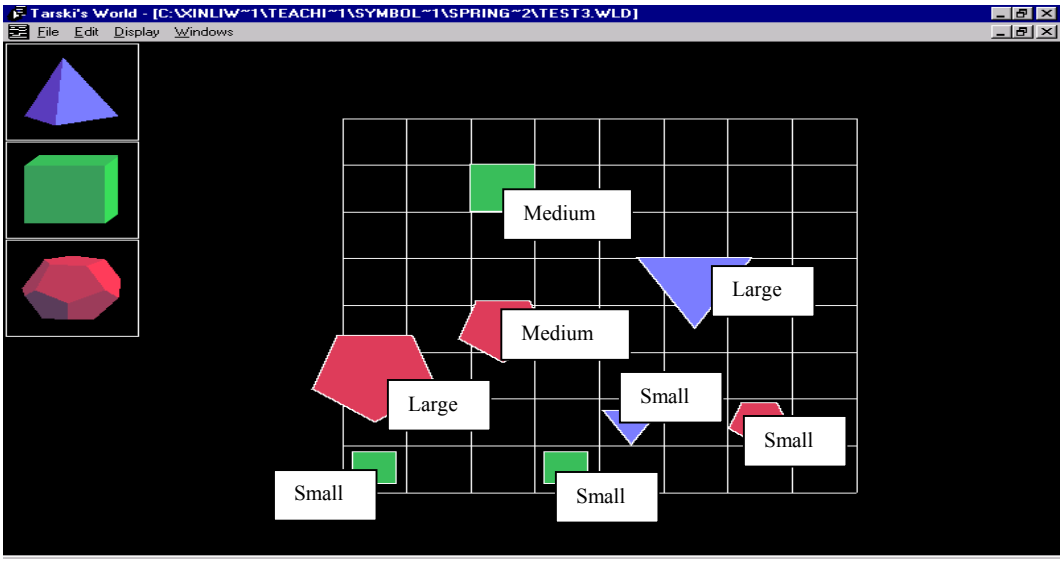

3.4. Solutions to Tests

3.4.1. Test One Solutions

Problem 1.

$\neg (A \vee \neg (B \wedge C)) \wedge \neg (\neg B \vee (A \vee B))$
$\Leftrightarrow \neg A \wedge \neg\neg (B \wedge C) \wedge \neg\neg B \wedge \neg (A \vee B)$
$\Leftrightarrow \neg A \wedge B \wedge C \wedge B \wedge \neg A \wedge \neg B$
$\Leftrightarrow \neg A \wedge B \wedge C \wedge \neg B$
$\Leftrightarrow (C \wedge B) \wedge (\neg B \wedge \neg A)$
$\Leftrightarrow (C \wedge B) \wedge \neg (B \vee A)$

Problem 2.
A translation manual

	English	FOL
Names	Max, Claire, Jenny, Nancy	same
Predicates	x marry y x is y x is in love with y x is intelligent.	Marry (x, y) x = y InLove (x, y) Intelligent (x)
Functions	the oldest daughter of x the youngest daughter of x	o-daughter (x) y-daughter (x)

Translations

(1) [Marry (Max, o-daughter(Nancy)) $\vee$ Marry (Max, y-daughter (Nancy))] $\wedge$
$\neg$ [Marry (Max, o-daughter(Nancy)) $\wedge$ Marry (Max, y-daughter (Nancy))] exclusive sense of OR!

(2) Jenny = y-daughter (Nancy) $\wedge$ Claire = y-daughter (Nancy)

(3) $\neg$ InLove(Claire, amx) $\wedge \neg$ InLOve(Jenny, Max) complete denial!

(4) ¬Marry (Jenny, Max) v (Intelligent (Max) ∧ InLove(Max, Jenny)) — the easiest!

OR

¬(Intelligent (Max) ∧ InLOve(Max, Jenny)) → ¬Marry (Jenny, Max) — closer to common sense!

OR

Marry (Jenny, Max) → (Intelligent (Max) ∧ InLove(Max, Jenny)) — closer to logical sense!

(5) ¬ (Intelligent(Max) ∧ InLOve(Max, Jenny)) — partial denial!

Problem 3.

(1) { (A ∧ C) v (D ∧ B) } |= C v B

1.	(A ∧ C) v (D ∧ B)	
2.	A ∧ C	
3.	C	∧Elim: 2
4.	C ∨ B	∨Intro: 3
5.	D ∧ B	
6.	B	∧Elim: 5
7.	C ∨ B	∨Intro: 6
8.	C ∨ B	∨Elim: 2-4, 5-7

(2) { (A ∧ C) v (D ∧ C), B } |= C ∧ B

1.	(A ∧ C) v (D ∧ C)	
2.	B	
3.	A ∧ C	
4.	C	∧Elim: 3
5.	C ∧ B	∧Intro: 4, 2
6.	D ∧ C	
7.	C	∧Elim: 6
8.	C ∧ B	∧Intro: 7, 2
9.	C ∧ B	∨Elim: 1, 3-5, 6-8

(3) { A v ¬ B, ¬ A } |= ¬ B

1.	A v ¬ B	
2.	¬ A	
3.	¬ B	
4.	¬ B	Reit: 3
5.	A	
6.	B	
7.	A ∧ ¬A	∧Intro: 5, 2
8.	¬ B	¬Intro: 6-7
9.	¬ B	∨Elim: 1, 3-4, 5-8

(4) { ¬ (P v Q) } |= ¬ P ∧ ¬Q

1.	¬ (P v Q)	
2.	P	
3.	P v Q	v Intro: 2
4.	(P v Q) ∧ ¬ (P v Q)	∧Intro: 3, 1
5.	¬P	¬Intro: 2-4
6.	Q	
7.	P v Q	v Intro: 6
8.	(P v Q) ∧ ¬ (P v Q)	∧Intro: 7, 1
9.	¬ Q	¬Intro: 6-8
10.	¬P ∧¬Q	∧Intro: 5, 9

Bonus:
{(Small (a) ∧ Smaller (a, b)) v (Large (b) ∧ Smaller (a, b)), c = b} |= Smaller (a, c) ∧ c = b

1.	(Small (a) ∧ Smaller (a, b)) v (Large (b) ∧ Smaller (a, b))	
2.	c = b	
3.	Small (a) ∧ Smaller (a, b)	
4.	Smaller(a, b)	∧Elim: 3
5.	Large (b) ∧ Smaller (a, b)	
6.	Smaller(a, b)	∧Elim: 5
7.	Smaller(a, b)	∨Elim: 1, 3-4. 5-6
8.	c = c	Refl=
9.	b = c	Ind. Id: 8, 2
10.	Smaller(a, c)	Ind. Id.: 7, 9
11.	Smaller(a, c) ∧ c= b	∧Intro: 10, 2

3.4.2. Test Two Solutions

Problem 1

1. ¬(RightOf(b, d) ∨ LeftOf(b, d)) → (Cube(b) ∨ Cube(d))

2. ¬ ((Larger(b, c) ∨ Larger(c, b)) ↔ (Tet(b) ∧ Dodec(c))
   or [(Large(b)∧Large(c)) ∨ (Small(b) ∧ Small(c)) ∨ (Medium(b) ∨ Medium(c))] ↔ [Tet(b) ∧ Dodec(c)]

3. (Cube(a) ∧ Cube(c)) → [(Small(a) ∨ Small(c)) ∧ ¬(Small(a) ∧ Small(c))]

4. ∀x [(Cube(x) ∧ FrontOf(x, a)) → Larger(x, a)]

5. ∃x (Cube(b) ∧ Small(x) ∧ Dodec(x) ∧ FrontOf(b, x))

6. ¬∀x [(Medium(x) ∧ Tet(x)) → BackOf(a, x)] Partial denial!
   or ∃x [Medium(x) ∧ Tet(x) ∧ ¬ BackOf(a, x)]

7. ¬∃x (Cube(x) ∧ Between(b, a, x)) Complete denial!
   or ∀x [Cube(x) → ¬Between(b, a, x)]

8. ∀x (Smaller(x, b) → Cube(x))

9. ∀x (Larger(b, x) → Cube(x))

10. ∀x [(Cube(x) ∧ FrontOf(x, a)) → BackOf(x, b)] Cross-reference!

Problem 2.

¬ [∃y (Tet(y) ∧ Large (y)) ∧ ∀y (Tet(y) → ¬ Large (y))]
⇔ ¬∃y (Tet(y) ∧ Large (y)) ∨ ¬ ∀y (Tet(y) → ¬ Large (y))
⇔ ∀y¬(Tet(y) ∧ Large (y)) ∨ ∃y¬(Tet(y) → ¬ Large (y))
⇔ ∀y (¬Tet(y) ∨¬Large(y)) ∨ ∃y (Tet(y) ∧ ¬¬Large(y))
⇔ ∀y (Tet(y) → ¬Large(y)) ∨ ∃y (Tet(y) ∧ Large(y))
⇔ ∀x (Tet(x) → ¬Large(x)) ∨ ∃y (Tet(y) ∧ Large(y))

Problem 3

(1) All small cubes are in back of *a*.
(2) Some large dodecahedron is to the right of some/a cube.
(3) Not all cubes are to the left of *a*. Partial denial!
(4) No large dodecahedron is in back of *a*. Complete denial!

Problem 4.

(1)

1. ∀x (Cube(x) → Small(x))	
2. ∀x (Small(x) → BackOf(x, b))	
(e)	
3. Cube(e) → Small(e))	∀Elim: 1
4. Small(e) → BackOf(e, b))	∀Elim: 2
5. Cube(e)	
6. Small (e)	→Elim: 3, 5
7. BackOf(e, b)	→Elim: 4, 6
8. Cube(e) → BackOf(e, b)	→Intro: 5-7
9. ∀x (Cube(x) → BackOf(x, b))	∀Intro: (e)-8

(2)

1. ∀x (Small(x) → Cube(x))	
2. ∃x¬Cube(x)	
(b)	
3. ¬Cube(b)	
4. Small(b) → Cube(b)	∀Elim: 1
5. Small(b)	
6. Cube(b)	→Elim: 5, 4
7. Cube(b) ∧¬Cube(b)	∧Intro: 3, 6
8. ¬Small(b)	¬Intro: 5-7
9. ∃x ¬Small(x)	∃Intro: 8
10. ∃x ¬Small(x)	∃Elim: 2, (b)-9

3.4.3. Test Three Solutions

Problem 1

(1) ∃x∀y (x ≠ y → RightOf(x, y)] or ∃x∀y RightOf(x, y)

(2) ∀x∃y (RightOf(x, y) ∧ x ≠ y) or ∀x∃y RightOf(x, y)

(3) ∀x [Cube(x) → ∃y (Dodec(y) ∧ LeftOf(x, y))]

(4) ∀x [∀y(Cube(y) → FrontOf(x, y)) → (Tet(x) ∧ Small(x))]

(5) ∀x [¬∃y FrontOf(y, x) → Small(x)]

(6) ¬∀x [Cube(x) → ∃y (Tet(y) ∧ Small(y) ∧ FrontOf(x, y))]

(7) ∃x [Tet(x) ∧ ∃y (Cube(y) ∧ ¬ (Larger(x, y) ∨ Smaller(x, y)))]
or ∃x [Tet(x) ∧ ∃y (Cube(y) ∧ ¬ Smaller(x, y))]

(8) ∃x [Cube(x) ∧ ∀y (x ≠ y → Larger(x, y)]

(9) Cube(a) ∧ ∀x [(Cube(x) ∧ x ≠ a) → ∃y(Dodec(y) ∧ FrontOf(x, y))] ∧ ¬∃x [Dodec(x) ∧ FrontOf(a, x)]

(10) $\exists x\,[\neg Cube(x) \wedge \forall y\,(Tet(y) \rightarrow BackOf(x, y))] \wedge \forall x\,[Cube(x) \rightarrow \neg\forall y\,(Tet(y) \rightarrow BackOf(x, y))]$

(11) $\exists x \exists y\,[Cube(x) \wedge FrontOf(x, a) \wedge Cube(y) \wedge FrontOf(y, a) \wedge x \neq y\,]$

(12) $\forall x \forall y\,[(Small(x) \wedge Cube(x) \wedge Small(y) \wedge Cube(y)) \rightarrow y = x]$

(13) $\exists x\,\{Cube(x) \wedge FrontOf(x, b) \wedge \forall y\,[(Cube(y) \wedge FrontOf(y, b)) \rightarrow y = x]\}$

(14) $\exists x\,[Cube(x) \wedge \forall y\,(Cube(y) \rightarrow y = x) \wedge FrontOf(x, b)]$

(15) $\exists x\,[Cube(x) \wedge \forall y\,(Cube(y) \rightarrow y = x) \wedge \exists y\,(Dodec(y) \wedge FrontOf(x, y)) \wedge \exists y(Tet(y) \wedge BackOf(x, y))]$

Problem 2

(1) Some large block (the specific one) is in back of every small block. (True)

(2) Every small block is in front of some large block (some unspecified one). (True)

(3) No cube is to the left of all large tetrahedrons. (False)

(4) Not all cubes are in front of some dodecahedron. (True)

(5) There is at most one large tetrahedron. (True)

(6) There are exactly two tetrahedrons. (True)

(7) The tetrahedron is in back of some large dodecahedron. (False)

(8) There is exactly one tetrahedron in back of some large dodecahedron. (True)