

## SECTION 3: PRACTICE TESTS AND QUIZZES

This section contains actual exams and quizzes given during the Spring 2000 and Summer 2000 terms. They have been condensed to save space in this booklet.

### 3.1. Quizzes (Solutions follow in 3.2)

#### 3.1.1. Quiz One

**Problem 1:** Using the table given on p. 24, translate the following sentences into FOL [40 points]

1. Claire gave Silly to Max at 3:05 p.m.
2. Max was angry at 2:10 p.m.
3. Folly *belonged to* Max at 3:00 p.m.
4. 3:01 p.m. *is later than* 3:00 p.m.
5. Folly *was erased* by Max at 2 p.m.

**Problem 2:** Create your own translation manual (using as less predicates as possible), and translate the following English sentences into FOL [60 points – Translation Manual: 25 points, Translations: 35 points].

1. Claire is *a* good student in Symbolic Logic.
2. Claire is *the* best student of Class 2000.
3. Claire is taller than the girl friend of Max's best friend.
4. Max prefers Claire to John's girl friend.
5. John and Nancy's youngest son is shorter than their oldest daughter.
6. The sum of 2 and 6 is less than the next number of 11.

#### 3.1.2. Quiz Two

**Problem 1.** Give a *formal* proof of the sentence “Larger (c, d)” from the premises “Larger (b, a)”, “c = b”, and “a = d”. [20 points]

**Problem 2.** Use Double Negation rule, DeMorgen's rules and other Derivation Rules to prove that the following pair of sentences are logically equivalent. [30 points]

$$(1) \neg [(\neg A \wedge \neg \neg B) \vee \neg (A \vee C)] \Leftrightarrow (C \vee A) \wedge (\neg B \vee A)$$

$$(2) \neg [(\neg A \vee B) \vee \neg (A \wedge \neg (A \vee B))] \Leftrightarrow A \wedge \neg (A \vee B)$$

**Problem 3.** Translate the following English sentences into the formal language of *the Tarski's World* (50 points).

- (1) Either  $a$  is smaller than  $b$  or both  $a$  and  $b$  are larger than  $c$ .
- (2)  $a$  and  $b$  are both in front of  $c$ ; moreover, both are smaller than it.
- (3)  $c$  is neither between  $a$  and  $b$ , nor in front of either of them.
- (4) Neither  $d$  nor  $c$  is in front of either  $b$  or  $e$ .
- (5) **Only one** block, either  $a$  or  $b$ , is between  $c$  and  $d$ .
- (6) **Extra credit:** Although a small cube  $a$  is in front of a large dodecahedron  $b$ , the former is to the left of the latter unless  $a$  is between two tetrahedrons  $c$  and  $d$ .

### 3.1.3. Quiz Three

**Problem 1:** Using the names and predicates presented in Table 1 on page 23, translate the following into FOL (50 points = 10 points  $\times$  5).

1. If Folly belonged to Max at 2 p.m., then it was not blank 5 minute later.
2. Claire erased Folly at 2 p.m. if and only if Max did not erase Folly at 2 p.m..
3. Folly was blank at 2 p.m. if Claire or Max erased it at 1:55 p.m..
4. Folly was blank at 2 p.m. unless Claire did not erase it at 1:55 p.m..
5. Folly was not blank at 2 p.m. only if neither Claire nor Max erased it 10 minutes before.

**Problem 2:** Give formal proofs of the following (50 points = 25 points  $\times$  2).

1.  $\{ A \rightarrow B, A \rightarrow (B \rightarrow C), B \rightarrow (C \rightarrow D) \} \models A \rightarrow D$
2.  $\{ (F \wedge G) \leftrightarrow H, F \rightarrow G \} \models F \leftrightarrow H$

## 3.2. Solutions to Quizzes

### 3.2.1. Quiz One Solutions

**Problem 1**

1. Gave (Claire, Silly, Max, 3:05)
2. Angry (Max, 2:10)
3. Owen (Max, Folly, 3:00)
4. 3:00 < 3:01
5. Erased (Max, Folly, 2:00)

**Problem 2****A. Translation manual**

	English	FOL
Names	Claire, Max, John, Nancy Symbolic Logic, Class 2000 2, 6, 11	Claire, Max, John, Nancy SymLogic, Class2000 2, 6, 11
Functions	the best student of x the girl friend of x the best friend of x the youngest son of x and y the oldest daughter of x and y the sum of x and y the next number of x	bestS (x) girlF (x) bestF(x) youngestS(x, y) oldestD (x, y) sum (x, y) nextN(x)
Predicates	x is a good student in y x is y x is taller than y x prefers y to z x is less than y	GStudent(x, y) x = y Taller (x, y) Prefer (x, y, z) Less (x, y)

**B. Translations**

1. GStudent (Claire, SymLogic)
2. Claire = bestS (Class2000)
3. Taller (Claire, girlF (bestF (Max)))
4. Prefer (Max, Claire, girlF(John))
5. Taller (oldestD(John, Nancy), youngestS(John, Nancy))
6. Less (sum (2, 6), nextN(11))

**3.2.2. Quiz Two Solutions****Problem 1.**

Proof:

- |    |               |               |
|----|---------------|---------------|
| 1. | Larger (b, a) |               |
| 2. | c = b         |               |
| 3. | a = d         |               |
| 4. | Larger (b, d) | Ind. Id: 1, 3 |
| 5. | c = c         | Refl =        |
| 6. | b = c         | Ind. Id: 5, 2 |
| 7. | Larger (c, d) | Ind. Id: 4, 6 |

**Problem 2.**

$$(1) \neg [(\neg A \wedge \neg \neg B) \vee \neg (A \vee C)] \Leftrightarrow? \Rightarrow (C \vee A) \wedge (\neg B \vee A)$$

Proof:

$$\begin{aligned} & \neg [(\neg A \wedge \neg \neg B) \vee \neg (A \vee C)] \\ \Leftrightarrow & \neg(\neg A \wedge \neg \neg B) \wedge \neg \neg (A \vee C) \\ \Leftrightarrow & (\neg \neg A \vee \neg \neg \neg B) \wedge (A \vee C) \\ \Leftrightarrow & (A \vee \neg B) \wedge (A \vee C) \\ \Leftrightarrow & (C \vee A) \wedge (\neg B \vee A), \text{ as desired.} \end{aligned}$$

$$(2) \neg [ (\neg A \vee B) \vee \neg (A \wedge \neg (A \vee B)) ] \Leftrightarrow? A \wedge \neg (A \vee B)$$

Proof:

$$\begin{aligned} & \neg [ (\neg A \vee B) \vee \neg (A \wedge \neg (A \vee B)) ] \\ \Leftrightarrow & \neg(\neg A \vee B) \wedge \neg\neg(A \wedge \neg (A \vee B)) \\ \Leftrightarrow & (\neg\neg A \wedge \neg B) \wedge (A \wedge \neg (A \vee B)) \\ \Leftrightarrow & (A \wedge \neg B) \wedge A \wedge (\neg A \wedge \neg B) \\ \Leftrightarrow & \cancel{A} \wedge \neg B \wedge A \wedge \neg A \wedge \cancel{\neg B} \\ \Leftrightarrow & A \wedge \neg A \wedge \neg B \\ \Leftrightarrow & A \wedge \neg (A \vee B), \text{ as desired.} \end{aligned}$$

**Problem 3.** Translate the following English sentences into the formal language of *the Tarski's World* (50 points).

- (1)  $\text{Smaller}(a, b) \vee (\text{Larger}(a, c) \wedge \text{Larger}(b, c))$
- (2)  $(\text{FrontOf}(a, c) \wedge \text{FrontOf}(b, c)) \wedge (\text{Smaller}(a, c) \wedge \text{Smaller}(b, c))$
- (3)  $\neg \text{Between}(c, a, b) \wedge \neg (\text{FrontOf}(c, a) \vee \text{FrontOf}(c, b))$
- (4)  $\neg(\text{FrontOf}(d, b) \vee \text{FrontOf}(d, e)) \wedge \neg(\text{FrontOf}(c, b) \vee \text{FrontOf}(c, e))$
- (5)  $(\text{Between}(a, c, d) \vee \text{Between}(b, c, d)) \wedge \neg(\text{Between}(a, c, d) \wedge \text{Between}(b, c, d))$
- (6)  $[(\text{Small}(a) \wedge \text{Cube}(a) \wedge \text{Large}(b) \wedge \text{Dodec}(b) \wedge \text{FrontOf}(a, b))] \wedge [\text{LeftOf}(a, b) \vee (\text{Tet}(c) \wedge \text{Tet}(d) \wedge \text{Between}(a, c, d))]$

### 3.2.3. Quiz Three Solutions

#### Problem 1

1. *If Folly belonged to Max at 2 p.m., then it was not blank 5 minute later.*  
 $\text{Owned}(\text{Max}, \text{Folly}, 2:00) \rightarrow \neg \text{Blank}(\text{Folly}, 2:05)$
  2. *Claire erased Folly at 2 p.m. if and only if Max did not erase Folly at 2 p.m..*  
 $\text{Erased}(\text{Claire}, \text{Folly}, 2:00) \leftrightarrow \neg \text{Erased}(\text{Max}, \text{Folly}, 2:00)$
  3. *Folly was blank at 2 p.m. if Claire or Max erased it at 1:55 p.m.*  
 $(\text{Erased}(\text{Claire}, \text{Folly}, 1:55) \vee \text{Erased}(\text{Max}, \text{Folly}, 1:55)) \rightarrow \text{Blank}(\text{Folly}, 2:00)$
  4. *Folly was blank at 2 p.m. unless Claire did not erase it at 1:55 p.m..*  
 $\text{Blank}(\text{Folly}, 2:00) \vee \neg \text{Erased}(\text{Claire}, \text{Folly}, 1:55)$
- Or  $\text{Erased}(\text{Claire}, \text{Folly}, 1:55) \rightarrow \text{Blank}(\text{Folly}, 2:00)$
5. *Folly was not blank at 2 p.m. only if neither Claire nor Max erased it 10 minutes before.*  
 $\neg \text{Blank}(\text{Folly}, 2:00) \rightarrow \neg (\text{Erased}(\text{Claire}, \text{Folly}, 1:50) \vee \text{Erased}(\text{Max}, \text{Folly}, 1:50))$

#### Problem 2

1.  $\{ A \rightarrow B, A \rightarrow (B \rightarrow C), B \rightarrow (C \rightarrow D) \} \models A \rightarrow D$

1.	$A \rightarrow B$	
2.	$A \rightarrow (B \rightarrow C)$	
3.	$B \rightarrow (C \rightarrow D)$	
	4.	
	5.	$B$ $\rightarrow$ Elim: 4, 1
	6.	$B \rightarrow C$ $\rightarrow$ Elim: 4, 2
	7.	$C \rightarrow D$ $\rightarrow$ Elim: 5, 3
	8.	$C$ $\rightarrow$ Elim: 5, 6
	9.	$D$ $\rightarrow$ Elim: 8, 7
10.	$A \rightarrow D$	$\rightarrow$ Intro: 4-9

2.  $\{ (F \wedge G) \leftrightarrow H, F \rightarrow G \} \models F \leftrightarrow H$

1.	$(F \wedge G) \leftrightarrow H$	
2.	$F \rightarrow G$	
3.	$F$	
4.	$G$	$\rightarrow$ Elim: 3, 2
5.	$F \wedge G$	$\wedge$ Intro: 3, 4
6.	$H$	$\leftrightarrow$ Elim: 5, 1
7.	$H$	
8.	$F \wedge G$	$\leftrightarrow$ Elim: 7, 1
9.	$F$	$\wedge$ Elim: 8
10.	$H \leftrightarrow G$	$\leftrightarrow$ Intro: 3-6, 7-9.

### 3.3. Tests (Solutions follow in 3.4)

#### 3.3.1. Test One

**Problem 1:** Use Double Negation rule, DeMorgan rules and any other rules to prove that the following pair of sentences are logically equivalent (20 points).

$$\neg (A \vee \neg (B \wedge C)) \wedge \neg (\neg B \vee (A \vee B)) \Leftrightarrow (C \wedge B) \wedge \neg (B \vee A)$$

**Problem 2:** By creating your own translation manual, translate the following English sentences into FOL (40 points).

- (1) Max can marry either Nancy's oldest daughter or her youngest daughter (in a monogamy society).
- (2) Jenny is Nancy's youngest daughter and Claire is her oldest daughter.
- (3) Neither Claire nor Jenny is in love with Max.
- (4) Jenny will not marry Max unless he is intelligent and in love with her.
- (5) Max is not both intelligent and in love with Jenny.

**Problem 3:** Give formal proofs of the followings (40 points).

- (1)  $\{ (A \wedge C) \vee (D \wedge B) \} \models C \vee B$  (about 8 steps)
- (2)  $\{ (A \wedge C) \vee (D \wedge C), B \} \models C \wedge B$  (about 10 steps)
- (3)  $\{ A \vee \neg B, \neg A \} \models \neg B$  (about 9 steps)
- (4)  $\{ \neg (P \vee Q) \} \models \neg P \wedge \neg Q$  (about 10 steps)
- (5) **Bonus** (up to 5 points)  
 $\{ (\text{Small}(a) \wedge \text{Smaller}(a, b)) \vee (\text{Large}(b) \wedge \text{Smaller}(a, b)), c = b \} \models \text{Smaller}(a, c) \wedge c = b$  (about 11 steps)

### 3.3.2. Test Two

**Problem 1.** Translate the following English sentences into FOL (using the language of Tarski's World; Domain: all the blocks in a Tarski's world). (40 points)

1. If  $b$  is neither to the right nor left of  $d$ , then at least one of them is a cube.
2.  $b$  and  $c$  are the same size if and only if  $b$  is a tetrahedron and  $c$  is a dodecahedron.
3.  $a$  and  $c$  are both cubes only if exactly one of them is small.
4. Any cube in front of  $a$  is larger than  $a$ .
5. Cube  $b$  is in front of some small dodecahedron.
6.  $a$  is not in back of every medium tetrahedron.
7.  $b$  is not between  $a$  and any cubes.
8. Only cubes are smaller than  $b$ .
9.  $b$  is larger than nothing but cubes.
10. If some cube is in front of  $a$ , then it is in back of  $b$ .

**Problem 2.** Prove the following pair of sentences are logically equivalent (14 points).

$$\neg [\exists y (\text{Tet}(y) \wedge \text{Large}(y)) \wedge \forall y (\text{Tet}(y) \rightarrow \neg \text{Large}(y))] \\ \Leftrightarrow \forall x (\text{Tet}(x) \rightarrow \neg \text{Large}(x)) \vee \exists y (\text{Tet}(y) \wedge \text{Large}(y))$$

**Problem 3.** For the following sentences in FOL, translate them into colloquial English sentences (16 points)

- (1)  $\forall x [ (\text{Small}(x) \wedge \text{Cube}(x)) \rightarrow \text{BackOf}(x, a) ]$
- (2)  $\exists x [ (\text{Dodec}(x) \wedge \text{Large}(x)) \wedge \exists y (\text{Cube}(y) \wedge \text{RightOf}(x, y)) ]$
- (3)  $\neg \forall x ( \text{Cube}(x) \rightarrow \text{LeftOf}(x, a) )$
- (4)  $\neg \exists x [ \text{Dodec}(x) \wedge \text{Large}(x) \wedge \text{BackOf}(x, a) ]$

**Problem 4.** Give formal proofs of the followings (30 points)

- (1)  $\{ \forall x (\text{Cube}(x) \rightarrow \text{Small}(x)), \quad \forall x (\text{Small}(x) \rightarrow \text{BackOf}(x, b)) \} \vdash \forall x (\text{Cube}(x) \rightarrow \text{BackOf}(x, b))$
- (2)  $\{ \forall x (\text{Small}(x) \rightarrow \text{Cube}(x)), \quad \exists x \neg \text{Cube}(x) \} \vdash \exists x \neg \text{Small}(x)$

### 3.3.3. Test Three

**Problem 1:** Translate the following English sentences into *the language of Tarski's World*.

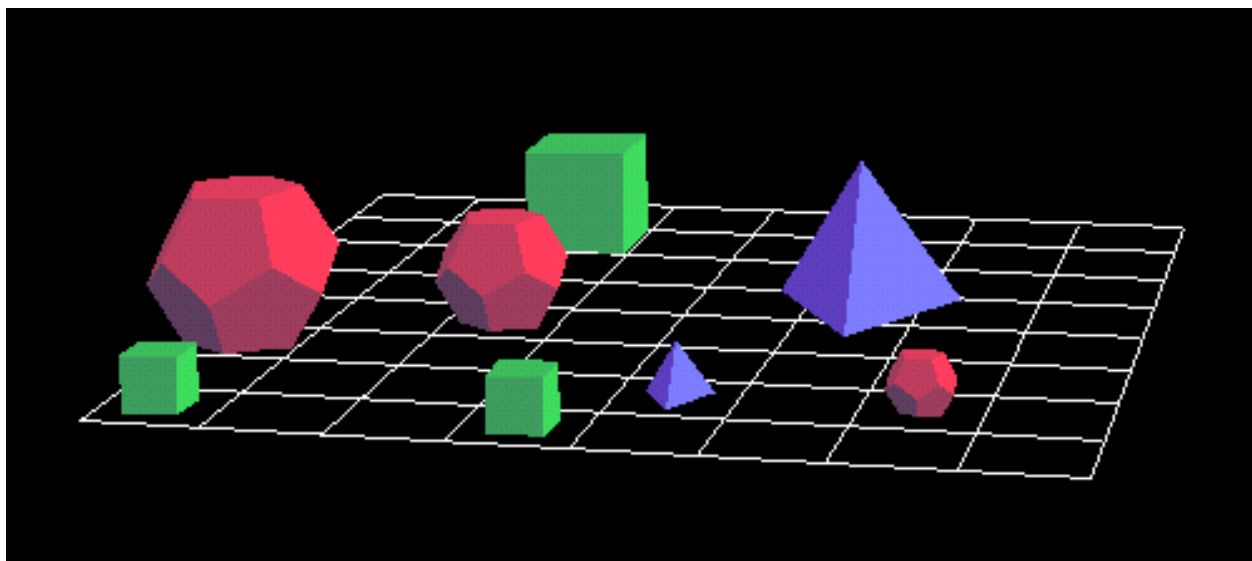
DD: all blocks in a Tarski's world. (60 points =  $15 \times 4$ )

- (1) Some block is to the right of every other block.
- (2) Every block is to the right of some other block.
- (3) If some block is a cube, then it is to the left of some dodecahedron.
- (4) *Only* small tetrahedrons are in front of all cubes.
- (5) Anything with *nothing* in front of it is small.

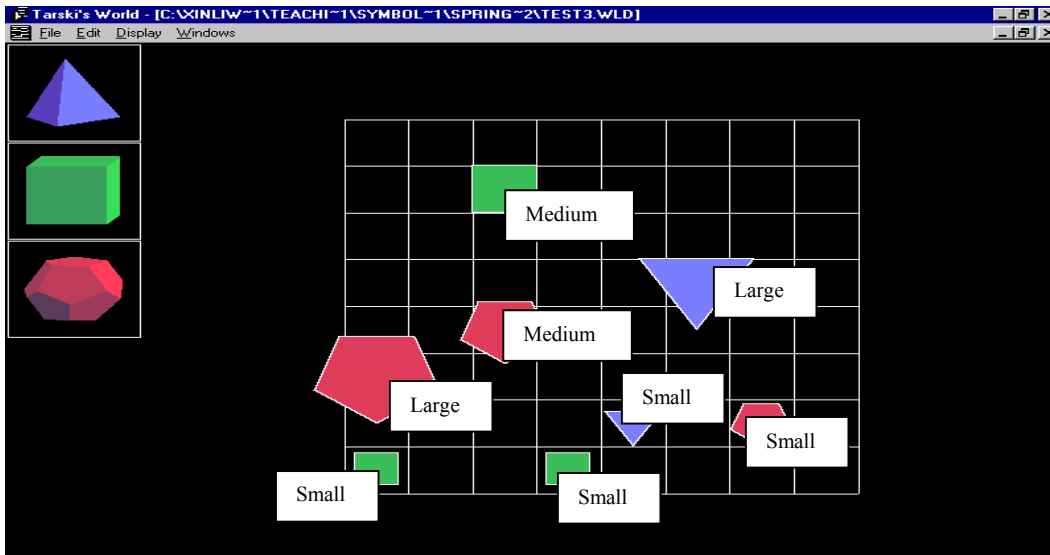
- (6) Not all cubes are in front of some small tetrahedron.
- (7) Some tetrahedron is as large as some cube.
- (8) Some cube is the largest block.
- (9) All cubes but  $a$  are in front of some dodecahedron.
- (10) Some block but cubes is in back of all tetrahedrons.
- (11) There are at least two cubes in front of  $a$ .
- (12) There is at most one small cube.
- (13) There is exactly one cube in front of  $b$ .
- (14) There is exactly one cube and it is in front of  $b$ .
- (15) The cube is in front of a dodecahedron and in back of a tetrahedron.

**Problem 2:** For the following sentences in FOL, (a) translate them into colloquial English sentences, and (b) assess their truth values on the basis of the world attached ( $40 \text{ points} = 8 \times 5$ ).

- \_\_\_\_\_ (1)  $\exists x \forall y [ \text{Large}(x) \wedge (\text{Small}(y) \rightarrow \text{BackOf}(x, y) ) ]$
- \_\_\_\_\_ (2)  $\forall y \exists x [ \text{Small}(y) \rightarrow (\text{Large}(x) \wedge \text{FrontOf}(y, x)) ]$
- \_\_\_\_\_ (3)  $\neg \exists x \{ \text{Cube}(x) \wedge \forall y [ (\text{Tet}(y) \wedge \text{Large}(y)) \rightarrow \text{LeftOf}(x, y) ] \}$
- \_\_\_\_\_ (4)  $\neg \forall x [ \text{Cube}(x) \rightarrow \exists y (\text{Dodec}(y) \wedge \text{FrontOf}(x, y)) ]$
- \_\_\_\_\_ (5)  $\forall x \forall y [ (\text{Tet}(x) \wedge \text{Tet}(y) \wedge \text{Large}(x) \wedge \text{Large}(y)) \rightarrow y = x ]$
- \_\_\_\_\_ (6)  $\exists x \exists y [ \text{Tet}(x) \wedge \text{Tet}(y) \wedge x \neq y \wedge \forall z (\text{Tet}(z) \rightarrow (z = x \vee z = y)) ]$
- \_\_\_\_\_ (7)  $\exists x [ \text{Tet}(x) \wedge \forall y (\text{Tet}(y) \rightarrow y = x) \wedge \exists z (\text{Large}(z) \wedge \text{Dodec}(z) \wedge \text{BackOf}(x, z)) ]$
- \_\_\_\_\_ (8)  $\exists x \{ \text{Tet}(x) \wedge \exists y (\text{Large}(y) \wedge \text{Dodec}(y) \wedge \text{BackOf}(x, y)) \wedge \forall z [ (\text{Tet}(z) \wedge \exists y (\text{Large}(y) \wedge \text{Dodec}(y) \wedge \text{BackOf}(z, y))) \rightarrow z = x ] \}$



(Be sure to look at the next page for a labeled image...)



### 3.4. Solutions to Tests

#### 3.4.1. Test One Solutions

**Problem 1.**

$$\neg (A \vee \neg (B \wedge C)) \wedge \neg (\neg B \vee (A \vee B))$$

$$\Leftrightarrow \neg A \wedge \neg \neg (B \wedge C) \wedge \neg \neg B \wedge \neg (A \vee B)$$

$$\Leftrightarrow \neg A \wedge B \wedge C \wedge B \wedge \neg A \wedge \neg B$$

$$\Leftrightarrow \neg A \wedge B \wedge C \wedge \neg B$$

$$\Leftrightarrow (C \wedge B) \wedge (\neg B \wedge \neg A)$$

$$\Leftrightarrow (C \wedge B) \wedge \neg (B \vee A)$$

**Problem 2.**

*A translation manual*

	English	FOL
Names	Max, Claire, Jenny, Nancy	same
Predicates	x marry y x is y x is in love with y x is intelligent.	Marry (x, y) x = y InLove (x, y) Intelligent (x)
Functions	the oldest daughter of x the youngest daughter of x	o-daughter (x) y-daughter (x)

*Translations*

- (1)  $[Marry (Max, o\text{-daughter}(Nancy)) \vee Marry (Max, y\text{-daughter} (Nancy))] \wedge \neg [Marry (Max, o\text{-daughter}(Nancy)) \wedge Marry (Max, y\text{-daughter} (Nancy))]$  exclusive sense of OR!
- (2)  $Jenny = y\text{-daughter} (Nancy) \wedge Claire = y\text{-daughter} (Nancy)$
- (3)  $\neg InLove(Claire, amx) \wedge \neg InLOve(Jenny, Max)$  complete denial!

- (4)  $\neg \text{Marry}(\text{Jenny}, \text{Max}) \vee (\text{Intelligent}(\text{Max}) \wedge \text{InLove}(\text{Max}, \text{Jenny}))$  the easiest!  
       OR  
 $\neg(\text{Intelligent}(\text{Max}) \wedge \text{InLOve}(\text{Max}, \text{Jenny})) \rightarrow \neg \text{Marry}(\text{Jenny}, \text{Max})$  closer to common sense!  
       OR  
 $\text{Marry}(\text{Jenny}, \text{Max}) \rightarrow (\text{Intelligent}(\text{Max}) \wedge \text{InLove}(\text{Max}, \text{Jenny}))$  closer to logical sense!
- (5)  $\neg (\text{Intelligent}(\text{Max}) \wedge \text{InLOve}(\text{Max}, \text{Jenny}))$  partial denial!

**Problem 3.**

- (1)
- $\{ (A \wedge C) \vee (D \wedge B) \} \models C \vee B$

1.	$(A \wedge C) \vee (D \wedge B)$	
2.	$A \wedge C$	
3.	$C$	$\wedge \text{Elim: } 2$
4.	$C \vee B$	$\vee \text{Intro: } 3$
5.	$D \wedge B$	
6.	$B$	$\wedge \text{Elim: } 5$
7.	$C \vee B$	$\vee \text{Intro: } 6$
8.	$C \vee B$	$\vee \text{Elim: } 2-4, 5-7$

- (2)
- $\{ (A \wedge C) \vee (D \wedge C), B \} \models C \wedge B$

1.	$(A \wedge C) \vee (D \wedge C)$	
2.	$B$	
3.	$A \wedge C$	
4.	$C$	$\wedge \text{Elim: } 3$
5.	$C \wedge B$	$\wedge \text{Intro: } 4, 2$
6.	$D \wedge C$	
7.	$C$	$\wedge \text{Elim: } 6$
8.	$C \wedge B$	$\wedge \text{Intro: } 7, 2$
9.	$C \wedge B$	$\vee \text{Elim: } 1, 3-5, 6-8$

- (3)
- $\{ A \vee \neg B, \neg A \} \models \neg B$

1.	$A \vee \neg B$	
2.	$\neg A$	
3.	$\neg B$	
4.	$\neg B$	$\text{Reit: } 3$
5.	$A$	
6.	$B$	
7.	$A \wedge \neg A$	$\wedge \text{Intro: } 5, 2$
8.	$\neg B$	$\neg \text{Intro: } 6-7$
9.	$\neg B$	$\vee \text{Elim: } 1, 3-4, 5-8$

- (4)
- $\{ \neg (P \vee Q) \} \models \neg P \wedge \neg Q$

1.	$\neg (P \vee Q)$	
2.	$P$	
3.	$P \vee Q$	$\vee \text{Intro: } 2$
4.	$(P \vee Q) \wedge \neg (P \vee Q)$	$\wedge \text{Intro: } 3, 1$
5.	$\neg P$	$\neg \text{Intro: } 2-4$
6.	$Q$	
7.	$P \vee Q$	$\vee \text{Intro: } 6$
8.	$(P \vee Q) \wedge \neg (P \vee Q)$	$\wedge \text{Intro: } 7, 1$
9.	$\neg Q$	$\neg \text{Intro: } 6-8$
10.	$\neg P \wedge \neg Q$	$\wedge \text{Intro: } 5, 9$

Bonus:

$$\{(Small(a) \wedge Smaller(a, b)) \vee (Large(b) \wedge Smaller(a, b)), c = b\} \models Smaller(a, c) \wedge c = b$$

1.	$(Small(a) \wedge Smaller(a, b)) \vee (Large(b) \wedge Smaller(a, b))$	
2.	$c = b$	
3.	$Small(a) \wedge Smaller(a, b)$	
4.	$Smaller(a, b)$	$\wedge$ Elim: 3
5.	$Large(b) \wedge Smaller(a, b)$	
6.	$Smaller(a, b)$	$\wedge$ Elim: 5
7.	$Smaller(a, b)$	$\vee$ Elim: 1, 3-4, 5-6
8.	$c = c$	Refl=
9.	$b = c$	Ind. Id: 8, 2
10.	$Smaller(a, c)$	Ind. Id.: 7, 9
11.	$Smaller(a, c) \wedge c = b$	$\wedge$ Intro: 10, 2

### 3.4.2. Test Two Solutions

#### Problem 1

1.  $\neg(RightOf(b, d) \vee LeftOf(b, d)) \rightarrow (Cube(b) \vee Cube(d))$
2.  $\neg((Larger(b, c) \vee Larger(c, b)) \leftrightarrow (Tet(b) \wedge Dodec(c)))$   
or  $[(Large(b) \wedge Large(c)) \vee (Small(b) \wedge Small(c)) \vee (Medium(b) \vee Medium(c))] \leftrightarrow [Tet(b) \wedge Dodec(c)]$
3.  $(Cube(a) \wedge Cube(c)) \rightarrow [(Small(a) \vee Small(c)) \wedge \neg(Small(a) \wedge Small(c))]$
4.  $\forall x [(Cube(x) \wedge FrontOf(x, a)) \rightarrow Larger(x, a)]$
5.  $\exists x (Cube(b) \wedge Small(x) \wedge Dodec(x) \wedge FrontOf(b, x))$
6.  $\neg \forall x [(Medium(x) \wedge Tet(x)) \rightarrow BackOf(a, x)]$  Partial denial!  
or  $\exists x [Medium(x) \wedge Tet(x) \wedge \neg BackOf(a, x)]$
7.  $\neg \exists x (Cube(x) \wedge Between(b, a, x))$  Complete denial!  
or  $\forall x [Cube(x) \rightarrow \neg Between(b, a, x)]$
8.  $\forall x (Smaller(x, b) \rightarrow Cube(x))$
9.  $\forall x (Larger(b, x) \rightarrow Cube(x))$
10.  $\forall x [(Cube(x) \wedge FrontOf(x, a)) \rightarrow BackOf(x, b)]$  Cross-reference!

#### Problem 2.

$$\neg [ \exists y (Tet(y) \wedge Large(y)) \wedge \forall y (Tet(y) \rightarrow \neg Large(y)) ]$$

$$\Leftrightarrow \neg \exists y (Tet(y) \wedge Large(y)) \vee \neg \forall y (Tet(y) \rightarrow \neg Large(y))$$

$$\Leftrightarrow \forall y \neg (Tet(y) \wedge Large(y)) \vee \exists y \neg (Tet(y) \rightarrow \neg Large(y))$$

$$\Leftrightarrow \forall y (\neg Tet(y) \vee \neg Large(y)) \vee \exists y (Tet(y) \wedge \neg \neg Large(y))$$

$$\Leftrightarrow \forall y (Tet(y) \rightarrow \neg Large(y)) \vee \exists y (Tet(y) \wedge Large(y))$$

$$\Leftrightarrow \forall x (Tet(x) \rightarrow \neg Large(x)) \vee \exists y (Tet(y) \wedge Large(y))$$

**Problem 3**

- (1) All small cubes are in back of  $a$ .
- (2) Some large dodecahedron is to the right of some/a cube.
- (3) Not all cubes are to the left of  $a$ .
- (4) No large dodecahedron is in back of  $a$ .

Partial denial!  
Complete denial!

**Problem 4.**

- (1)
- |  |                          |
|--|--------------------------|
| 1. $\forall x (Cube(x) \rightarrow Small(x))$      |                          |
| 2. $\forall x (Small(x) \rightarrow BackOf(x, b))$ |                          |
| (e)  |                          |
| 3. $Cube(e) \rightarrow Small(e)$                  | $\forall$ Elim: 1        |
| 4. $Small(e) \rightarrow BackOf(e, b)$             | $\forall$ Elim: 2        |
| 5. $Cube(e)$                                       |                          |
| 6. $Small(e)$                                      | $\rightarrow$ Elim: 3, 5 |
| 7. $BackOf(e, b)$                                  | $\rightarrow$ Elim: 4, 6 |
| 8. $Cube(e) \rightarrow BackOf(e, b)$              | $\rightarrow$ Intro: 5-7 |
| 9. $\forall x (Cube(x) \rightarrow BackOf(x, b))$  | $\forall$ Intro: (e)-8   |

- (2)
- |   |                          |
|---|--------------------------|
| 1. $\forall x (Small(x) \rightarrow Cube(x))$ |                          |
| 2. $\exists x \neg Cube(x)$                   |                          |
| (b)   |                          |
| 3. $\neg Cube(b)$                             |                          |
| 4. $Small(b) \rightarrow Cube(b)$             | $\forall$ Elim: 1        |
| 5. $Small(b)$                                 |                          |
| 6. $Cube(b)$                                  | $\rightarrow$ Elim: 5, 4 |
| 7. $Cube(b) \wedge \neg Cube(b)$              | $\wedge$ Intro: 3, 6     |
| 8. $\neg Small(b)$                            | $\neg$ Intro: 5-7        |
| 9. $\exists x \neg Small(x)$                  | $\exists$ Intro: 8       |
| 10. $\exists x \neg Small(x)$                 | $\exists$ Elim: 2, (b)-9 |

**3.4.3. Test Three Solutions****Problem 1**

- (1)  $\exists x \forall y (x \neq y \rightarrow RightOf(x, y))$       or       $\exists x \forall y RightOf(x, y)$
- (2)  $\forall x \exists y (RightOf(x, y) \wedge x \neq y)$       or       $\forall x \exists y RightOf(x, y)$
- (3)  $\forall x [Cube(x) \rightarrow \exists y (Dodec(y) \wedge LeftOf(x, y)) ]$
- (4)  $\forall x [\forall y (Cube(y) \rightarrow FrontOf(x, y)) \rightarrow (Tet(x) \wedge Small(x)) ]$
- (5)  $\forall x [\neg \exists y FrontOf(y, x) \rightarrow Small(x) ]$
- (6)  $\neg \forall x [Cube(x) \rightarrow \exists y (Tet(y) \wedge Small(y) \wedge FrontOf(x, y)) ]$
- (7)  $\exists x [Tet(x) \wedge \exists y (Cube(y) \wedge \neg (Larger(x, y) \vee Smaller(x, y))) ]$   
or  $\exists x [Tet(x) \wedge \exists y (Cube(y) \wedge \neg Smaller(x, y)) ]$
- (8)  $\exists x [Cube(x) \wedge \forall y (x \neq y \rightarrow Larger(x, y)) ]$
- (9)  $Cube(a) \wedge \forall x [(Cube(x) \wedge x \neq a) \rightarrow \exists y (Dodec(y) \wedge FrontOf(x, y))] \wedge \neg \exists x [Dodec(x) \wedge FrontOf(a, x)]$

- (10)  $\exists x [\neg \text{Cube}(x) \wedge \forall y (\text{Tet}(y) \rightarrow \text{BackOf}(x, y))] \wedge \forall x [\text{Cube}(x) \rightarrow \neg \forall y (\text{Tet}(y) \rightarrow \text{BackOf}(x, y))]$
- (11)  $\exists x \exists y [\text{Cube}(x) \wedge \text{FrontOf}(x, a) \wedge \text{Cube}(y) \wedge \text{FrontOf}(y, a) \wedge x \neq y]$
- (12)  $\forall x \forall y [(\text{Small}(x) \wedge \text{Cube}(x) \wedge \text{Small}(y) \wedge \text{Cube}(y)) \rightarrow y = x]$
- (13)  $\exists x \{ \text{Cube}(x) \wedge \text{FrontOf}(x, b) \wedge \forall y [(\text{Cube}(y) \wedge \text{FrontOf}(y, b)) \rightarrow y = x] \}$
- (14)  $\exists x [\text{Cube}(x) \wedge \forall y (\text{Cube}(y) \rightarrow y = x) \wedge \text{FrontOf}(x, b)]$
- (15)  $\exists x [\text{Cube}(x) \wedge \forall y (\text{Cube}(y) \rightarrow y = x) \wedge \exists y (\text{Dodec}(y) \wedge \text{FrontOf}(x, y)) \wedge \exists y (\text{Tet}(y) \wedge \text{BackOf}(x, y))]$

### Problem 2

- (1) Some large block (the specific one) is in back of every small block. (True)
- (2) Every small block is in front of some large block (some unspecified one). (True)
- (3) No cube is to the left of all large tetrahedrons. (False)
- (4) Not all cubes are in front of some dodecahedron. (True)
- (5) There is at most one large tetrahedron. (True)
- (6) There are exactly two tetrahedrons. (True)
- (7) The tetrahedron is in back of some large dodecahedron. (False)
- (8) There is exactly one tetrahedron in back of some large dodecahedron. (True)